Section 16.5 - Curl and Divergence

Recall: Two operations of vectors ; Two properties of a vector

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★ Want to talk about two rates of change for vectors

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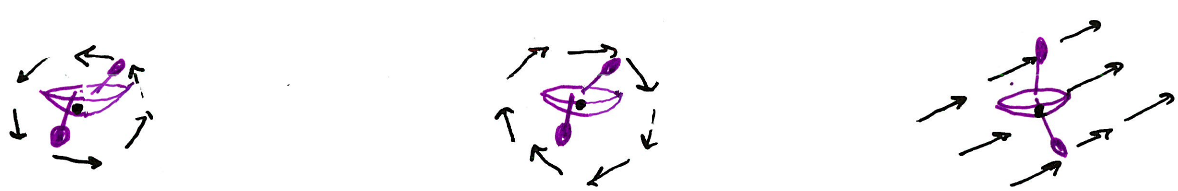
• Curl: $\vec{F} = \langle P, Q, R \rangle$ on $\mathbb{R}^3$, Partials of P, Q, R exist then:

• Understanding Curl with paddle boats:



★ Work on Vector field worksheet

Do they have positive, negative or zero curl?

If Curl $\vec{F} = \vec{0}$, $\vec{F}$ is called

Section 16.5 – Curl and Divergence

MVC

Theorem If f is a function of 3 variables with continuous second order partial derivatives then: $\text{Curl}(\nabla f) =$

Proof:

★ $\vec{F}$ conservative $\Rightarrow$

Theorem (Partial converse to above statement)

$\vec{F}$ defined on , components have continuous first partials

and then

Example (a) Show $\vec{F} = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$ is conservative.

(b) Find f so that $\nabla f = \vec{F}$.

• Green's Theorem Rewritten: $\int_C P\,dx + Q\,dy = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA$

$\vec{F} = \langle P, Q, 0 \rangle$ $\quad \text{Curl}(\vec{F}) =$

Section 16.5 - Curl and Divergence

- **Divergence**: $\vec{F} = \langle P, Q, R \rangle$ defined on $\mathbb{R}^3$, Partials of P, Q, R exist then:

- Understanding Divergence:

★ Work on Vector field worksheet

If $\operatorname{div} \vec{F} = 0$, $\vec{F}$ is called

Do they have positive, negative or zero divergence?

Theorem $\vec{F} = \langle P, Q, R \rangle$ on $\mathbb{R}^3$, P, Q, R have continuous second partials then

$$\operatorname{Div}(\operatorname{Curl} \vec{F}) =$$

Proof:

Example Show $\vec{F} = \langle xz, xyz, -y^2 \rangle$ can't be written as the curl of another vector field, that is $\vec{F} \neq \operatorname{Curl} \vec{G}$ for any $\vec{G}$.

- Laplace Operator:

$$\operatorname{div}(\nabla f) =$$

Laplace Equation:

★ Watch Water flow video on Website