Section 16.3 - Fundamental Theorem for Line Integrals

MVC

- **Recap:** Part 1:

  where $C: \vec{r}(t)$, $a \le t \le b$ is piecewise smooth
  f differentiable, ∇f continuous on C

  Part 2:

  for $\vec{F}$ continuous on Open Connected D

- **Goal:** Showing $\int_C \vec{F} \cdot d\vec{r}$ is independent of path is hard
  Want an easier way to show $\vec{F}$ is conservative!

- **Idea:** Assume $\vec{F}$ is conservative work backwards to find conditions on $\vec{F}$.

  Suppose $\vec{F} = \langle P, Q \rangle$ is a conservative vector field
  that means:

  Assume P, Q have continuous first order partial Derivatives

  So by:

Theorem If $\vec{F}(x,y) = \langle P, Q \rangle$ is conservative where P, Q have continuous first order partial derivatives then:

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- Simply-Connected Regions:

  Simple Curve:

  Closed Curve:

  Simply-Connected region:

Simple + Not closed	Not simple + Not closed
Not Simple + Closed	Simple + Closed

Connected + Not Simply Connected	Not Connected + Not Simply Connected	Simply Connected

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Theorem (Partial Converse of the Last theorem)

$\vec{F} = \langle P, Q \rangle$ a vector field on an open Simply Connected region D, P, Q have continuous first order partial Derivatives with

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

throughout D then:

Proof:

Example Determine if $\vec{F}(x,y) = \langle 3 + 2xy,\ x^2 - 3y^2 \rangle$ is conservative. If it is find its potential function.

Example Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $C: \vec{r}(t) = \langle e^t \sin t,\ e^t \cos t \rangle$ $0 \le t \le \pi$ and $\vec{F}(x,y) = \langle 3 + 2xy,\ x^2 - 3y^2 \rangle$.

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- Conservation of Energy:

  $\vec{F}$ a force field moves an object of mass m along a curve $C: \vec{r}(t)$ $a \le t \le b$

  Newton's Second Law:

  Work done =

  Kinetic Energy of the object $K(t) =$

  Assume $\vec{F}$ is conservative so:

  Potential Energy of the object at (x,y,z) is defined by:

  By Fundamental Theorem for Line integrals we have:

  Work done =

  Law of Conservation of Energy: