Section 16.3 - Fundamental Theorem for Line Integrals

Recall: Fundamental Theorem of Calculus (FTC)

Part 1: F' continuous on $[a,b]$, $\int_a^b F'(x) =$

Part 2: $F(x) =$ where $\frac{d}{dx}(F(x)) = F'(x)$

Theorem FTC for Line Integrals Part 1:

C a smooth curve given by $\vec{r}(t)$, $a \le t \le b$, f differentiable with ∇f continuous on C then $\int_C \nabla f \cdot d\vec{r} =$

Proof:

★ Note:

- Conservative Vector Field:

★ Recall:

- Independence of Path:

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MVC

<u>Goal</u>: ① Want part 2 of FTC for Line Integrals - i.e. writing the potential function as a Line Integral.
② But how do we know we can? Need $\vec{F}$ path independent
③ Definition of Path Independence hard to check → Find easier way

- Closed curves:

A curve C is <u>closed</u> if

$\int_C \vec{F}\cdot d\vec{r}$ independent of path in D, C closed in D then

$$\int_C \vec{F}\cdot d\vec{r} =$$

$\boxed{\text{Theorem}}$ $\int_C \vec{F}\cdot d\vec{r}$ is independent of path in D iff

Open Not Connected

- Open Connected Regions:

D is <u>open</u> if

Connected Not open

D is <u>Connected</u> if

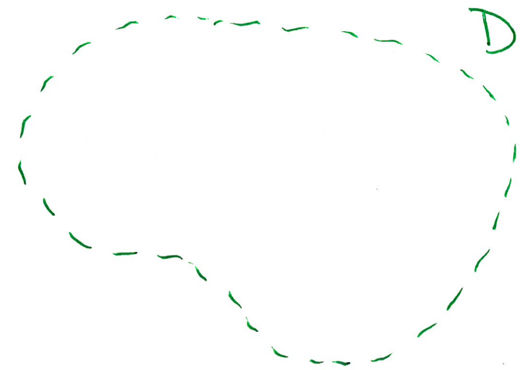
Open And connected

$\boxed{\text{Theorem}}$ Fundamental Theorem for Line Integrals Part 2:
$\vec{F}$ continuous on open connected region D.

Proof: FTC for Line Integrals part 2

Assume $\int_C \vec{F} \cdot d\vec{r}$ is independent of path in D

Show: There is f with $\nabla f = \vec{F}$ (means $\vec{F}$ is conservative)



$\boxed{\text{Example}}$ Let $f(x,y) = \sin(x-2y)$. Compute $\int_C \nabla f \cdot d\vec{r}$ where C is any curve that starts at $(0,0)$ and ends at $(\pi/3, \pi/2)$. Then find a curve not closed C_1 so that $\int_{C_1} \nabla f \cdot d\vec{r} = 0$.