Section 16.2 - Line Integrals in Space & of Vector Fields

MVC

- Line integral of f over C:

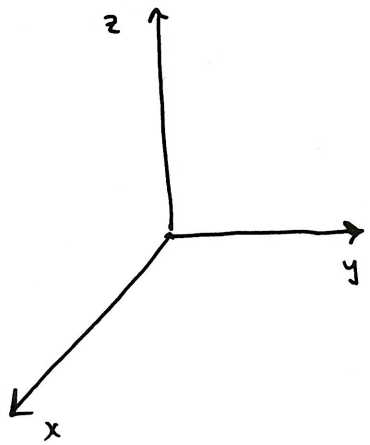
  C: $x = x(t)$, $y = y(t)$, $z = z(t)$, $a \le t \le b$

  wrt arc length: $\int_C f\, ds =$

  Similar wrt x, y, z

Example Evaluate $\int_C y\,dx + z\,dy + x\,dz$, where C consists of the lines C_1 from $(2,0,0)$ to $(3,4,5)$ followed by C_2 from $(3,4,5)$ to $(3,4,0)$.

- Line Integrals of Vector Fields: We will understand this by way of an application
  - Work done by force $F(x)$ in the x-direction from $x=a$ to $x=b$
  - Now suppose $\vec{F} = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$ is a continuous force field on $\mathbb{R}^3$. Compute work done to move a particle along curve C in $\mathbb{R}^3$.



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- Definition:

- Orientation Change:

$$\int_{-C} \vec{F} \cdot d\vec{r} =$$

- Notation:

$\vec{F} = \langle P, Q, R \rangle$ then $\int_C \vec{F} \cdot d\vec{r} =$

Example Find the work done by the force field $\vec{F}(x,y) = \langle x^2, -xy \rangle$ in moving a particle along $\vec{r}(t) = \langle \cos t, \sin t \rangle$ for $0 \le t \le \pi/2$.

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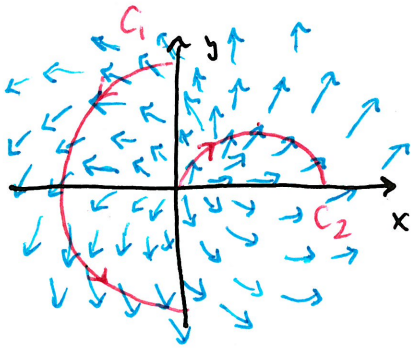
- Extra Examples

#18. Are the line integrals of $\vec{F}$ over C_1 and C_2 positive, negative or zero? Explain.



#21. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle \sin x, \cos y, xz \rangle$ $\vec{r}(t) = \langle t^3, -t^2, t \rangle$ $0 \le t \le 1$

#45. A 160-lb man carries 25-lb can of paint up a helical staircase that encircles a silo with a radius of 20 ft. If the silo is 90 ft tall and the man makes exactly 3 revolutions climbing to the top, find the work done by the man against gravity.