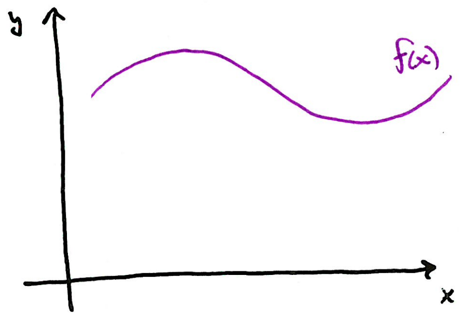
Section 16.2 - Line Integrals of Functions

MVC

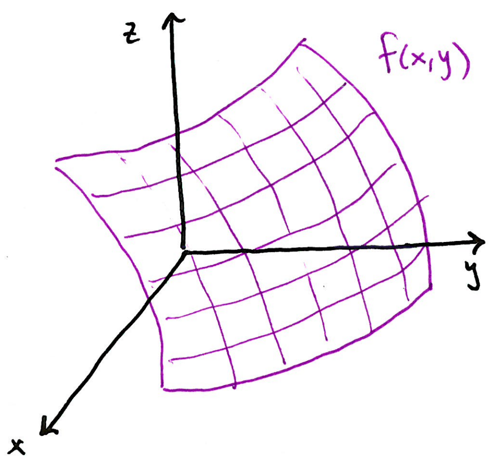
Goal: Integrate vector fields but before we do must understand Integration of functions first.

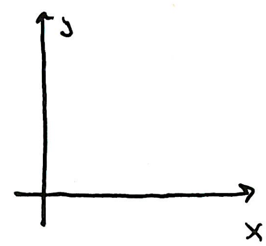
★ Ribbon of paper activity

- Line Integral in 2D



- Line Integral in 3D





- Line integral for f above C wrt arc length:

- Other Line integrals for f above C:

★ Visit Line integral demo on Website

Section 16.2 - Line Integrals of Functions

- Changing Direction: $-C$ means travel C backwards

$$\int_b^a f(x)\,dx = \qquad\qquad \int_{-C} f\,ds =$$

$$\int_{-C} f\,dx = \qquad\qquad \int_{-C} f\,dy =$$

- Properties:

Recall: $a<c<b$ then $\int_a^b f(x)\,dx =$

Similar: C piecewise-smooth union $C = C_1 \cup C_2$ then

$$\int_C f\,ds =$$

$\boxed{\text{Example}}$ Evaluate $\int_C 2x\,ds$ where C consists of C_1: $y=x^2$ from $(0,0)$ to $(1,1)$ and C_2: vertical line from $(1,1)$ to $(1,2)$.

- Notation:

$$\int_C P(x,y)\,dx + \int_C Q(x,y)\,dy =$$

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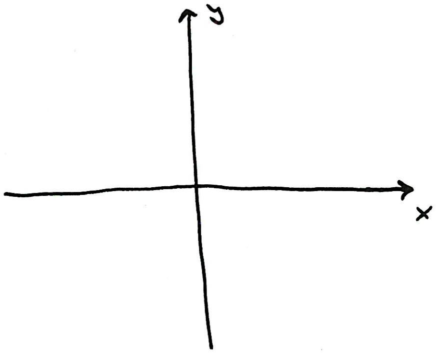
$\boxed{\text{Example}}$ Evaluate $\int_C y^2\,dx + x\,dy$, where (a) $C = C_1$ is the line segment from $(-5,-3)$ to $(0,2)$ and (b) $C = C_2$ is the arc $x = 4 - y^2$ from $(-5,-3)$ to $(0,2)$.



★ Conclusion:

- Application of Line Integrals:

  $f(x,y) = \rho(x,y)$ density function of a thin wire

  Mass of wire:

  Center of mass:

$\boxed{\text{Example}}$ A wire is in the shape of the semicircle $x^2+y^2=1$, $y \geq 0$ and is thicker near its base than near the top. Find the center of mass of the wire if the linear density at any point is proportional to its distance from $y=1$.