★ We used double integrals to compute the mass, center of mass and moments of laminas (thin plates) but what about solid objects in 3D?

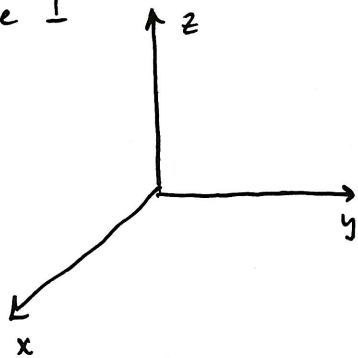Will need to add in 3 directions → Triple Integral

- Triple Integrals over a rectangular box:

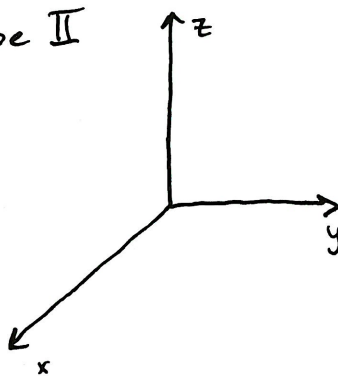$$B = \{(x,y,z) \mid a \leq x \leq b, \ c \leq y \leq d, \ e \leq z \leq f\}$$

- Fubini's Theorem for Triple Integrals:

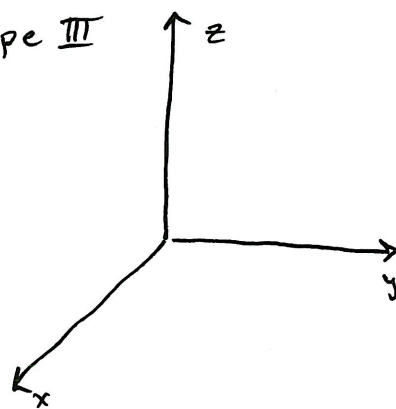- Triple Integrals over a 3D region:

Type I

Type II

Type III

Section 15.7 - Triple Integrals

Example Evaluate $\iiint\limits_E \sqrt{x^2 + z^2} \, dV$, where E is bounded by the paraboloid

$y = x^2 + z^2$ and the plane $z = 4$

Example Express the integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z) \, dz \, dy \, dx$ as a triple integral the

5 other ways.

• Applications of Triple Integrals:

E a closed region in 3D then: $V(E) =$

Mass of E with density function $f(x,y,z)$:

Moments about the coordinate planes:

Center of mass:

Moments of Inertia about the coordinate axes:

Section 15.7 - Triple Integrals

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- Extra Examples:

 # 19. Find the volume of the region bounded by $2x + y + z = 4$ and the coordinate planes as a triple integral.

 # 28. Sketch the solid whose volume is given by $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$.

#36. Write the 5 other iterated integrals for: $\int_0^1 \int_y^1 \int_0^z f(x, y, z)\, dx\, dz\, dy$