Section 15.5 - Applications of Double Integrals

MVC

Physical Applications: Computing Mass, electric charge, Center of mass, inertia

Already seen: Average value of a function $f_{ave} = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA$

Consider a thin plate (called a lamina) with variable density occupying a region D in the xy-plane:

- **Goals**: Given a density function $\rho(x,y)$ for a Lamina find:
  1.
  2.
  3.
  4.

- **Total mass of Lamina**: sum masses over ΔA

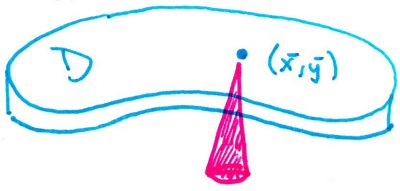
  In general density = but for thin plate density =

- **Moments of Lamina**: product of mass and its directed distance from an axis. (measures the tendency for the plate to rotate about the ax

  About x-axis:

  About y-axis:

- **Center of Mass**: point where plate balances horizontally
  ★ Point may not be on plate



★ See moments demo on website / Paper plate demo

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- **Moments of Inertia:**

  Mass determines force needed for an acceleration

  So Inertia determines torque needed for an angular acceleration

About x-axis:

About y-axis:

About origin:

Example The Density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass.

Example Find the moment of inertia I_0 of a homogeneous D with density ρ Center (0,0) and radius a. Use I_0 to determine I_x, I_y.

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- Extra Examples

#11. A lamina occupies the first quadrant of $x^2+y^2 \leq 1$. Find its center of mass if the density at any point is proportional to its distance from the x-axis.

#12. Lamina from #11 but the density at any point is proportional to the square of its distance from the origin.

#18. Find I_o, I_x, I_y for the lamina in #12.