Section 15.4 - Double Integrals in Polar Coordinates

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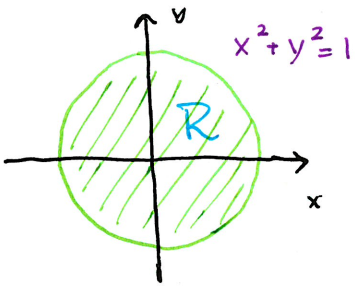
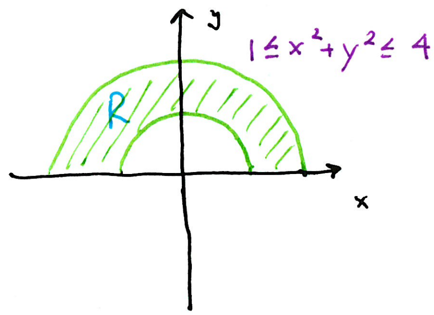
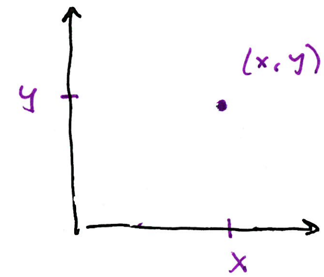
★ Regions that are circular in nature are difficult to describe in Cartesian Coordinates but are easy in Polar Coordinates.



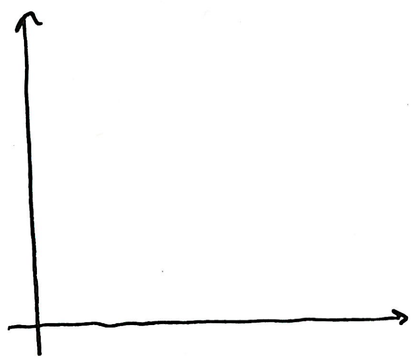




Rectangular: R =   R =

Polar: R =   R =

- Area of a small Polar Rectangle:



Important Identities:

①

②

③

- Change to Polar Coords in a Double Integral:

★ Watch demo on website

$\boxed{\text{Example}}$ Evaluate $\iint_R (3x+4y^2)\,dA$ where R is the region in the upper-half plane bounded by $x^2+y^2=1$ and $x^2+y^2=4$.

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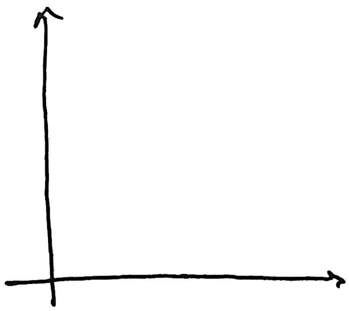
★ Polar regions but not polar rectangles:



$\boxed{\text{Example}}$ Use a double integral to find the area enclosed by one loop of the four-leaf rose curve: $r = \cos(2\theta)$.

$\boxed{\text{Example}}$ Find the volume of the solid that lies under $z = x^2 + y^2$ above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.

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- Extra Examples

#6. Sketch the region whose area is given by $\int_{\pi/2}^{\pi}\int_{0}^{2\sin\theta} r\,dr\,d\theta$.

#15. Find the area of one loop of $r=\cos(3\theta)$ using a double integral.

#25. Find the volume of the solid above the cone $z=\sqrt{x^2+y^2}$ and below $x^2+y^2+z^2=1$.

#39. Use polar coords to combine the sum into one double integral.

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\,dy\,dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,dy\,dx$$