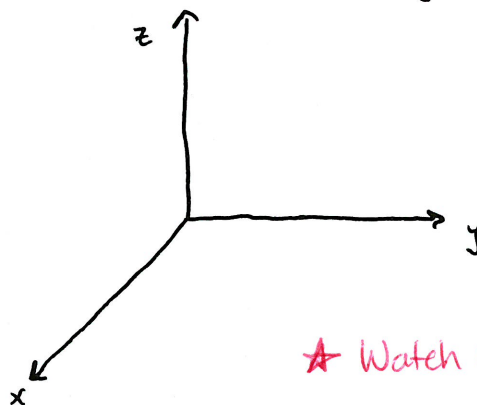
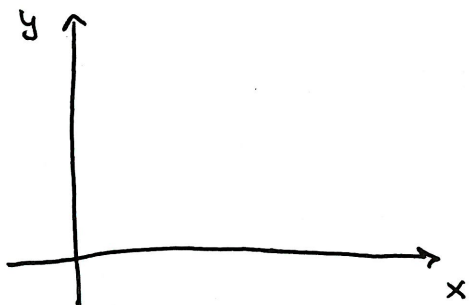


# Section 15.3 - Double Integrals over General Regions

MVC

★ Want to integrate over regions of a general shape

- Type I - Regions  $D = \{(x, y) \mid a \leq x \leq b, g(x) \leq y \leq h(x)\}$

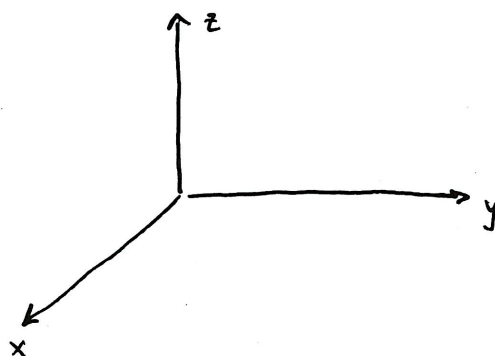
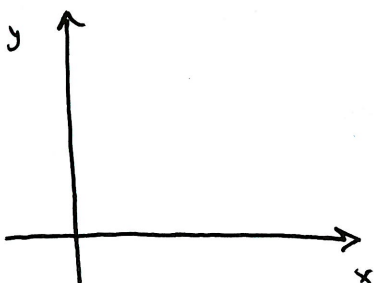


★ Watch demo on website

Fix  $x$ , find area of slice of  $z = f(x, y)$ :

Summing these areas as  $x$  varies on  $[a, b]$ :

- Type II - Regions  $D = \{(x, y) \mid c \leq y \leq d, g(y) \leq x \leq h(y)\}$



**Example**

Evaluate  $\iint_D (x+2y) dA$  where  $D$  is the region bounded by the parabolas  $y = 2x^2$  and  $y = 1+x^2$ .

## Section 15.3 - Double Integrals Over General Regions

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**Example** Evaluate  $\iint_D xy \, dA$ , where  $D$  is the region bounded by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ .

• Properties of Double Integrals:

①

②

③

④

⑤

⑥

$\frac{2}{3}$

## Section 15.3 - Double Integrals over General Regions

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### • Extra Examples

# 17.  $\iint_D x \cos y \, dA$ ,  $D$  is bounded by  $y=0$ ,  $y=x^2$ ,  $x=1$

# 21.  $\iint_D (2x-y) \, dA$ ,  $D$  bounded by circle at  $(0,0)$  with radius 2.

# 47. Sketch the region and reverse the order of integration  $\int_1^2 \int_0^{\ln x} f(x,y) \, dy \, dx$ .

# 62.  $\iint_D f(x,y) \, dA = \int_0^1 \int_0^{2y} f(x,y) \, dx \, dy + \int_1^3 \int_0^{3-y} f(x,y) \, dx \, dy$

Sketch  $D$  and reverse the order of Integration.