Section 15.2 - Iterated Integrals

MVC

★ In practice we do not evaluate single integrals by using the definition - we use Fundamental Theorem of Calculus (FTC)

For a fixed x on $R=[a,b]\times[c,d]$ we compute the area under $f(x,y)$ above $[c,d]$:

Suming up the areas as x varies over $[a,b]$ is the same as integrating A with respect to x:

Example Evaluate:

(a) $\int_0^3 \int_1^2 x^2 y \, dy \, dx$

(b) $\int_1^2 \int_0^3 x^2 y \, dx \, dy$

Fubini's Theorem If f is continuous on $R=[a,b]\times[c,d]$ then

Counter Example: $f(x,y) = \dfrac{x^2-y^2}{(x^2+y^2)^2}$ on $R=[0,1]\times[0,1]$

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$\boxed{\text{Example}}$ Evaluate $\iint_R y \sin(xy)\, dA$ where $R = [1,2] \times [0,\pi]$

- Double Integral as product of 2 Single Integrals:

- Integration Review:

① $\int x^n \, dx =$

all $n \neq -1$

② $\int x^{-1} \, dx =$

③ $\int e^x \, dx =$

④ $\int \frac{1}{1+x^2} \, dx =$

⑤ $\int \sin x \, dx =$

⑥ $\int \cos x \, dx =$

⑦ $\int \sec^2 x \, dx =$

⑧ $\int \sec x \tan x \, dx =$

- U-substitution: $\int g'(x) f'(g(x))\, dx =$

$u =$ $du =$

- Integration by Parts: $\int u\,dv =$

- Change of Coords: $\int_a^b f(x)\,dx =$

where $a =$ and $b =$

- Trig Substitution: $\sin^2 x + \cos^2 = 1$ $\tan^2 x + 1 = \sec^2 x$

$\sin^2 x = \frac{1 - \cos(2x)}{2}$ $\cos^2 x = \frac{1 + \cos(2x)}{2}$

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- Extra Examples:

#19. $\iint_R x\sin(x+y)\,dA$, $R=[0,\pi/6]\times[0,\pi/3]$

#20. $\iint_R \frac{x}{1+xy}\,dA$, $R=[0,1]\times[0,1]$

#35. Find the average value of $f(x,y)=x^2y$ over $[-1,1]\times[0,5]$

#38. Use symmetry to compute $\iint_R (1+x^2\sin y+y^2\sin x)\,dA$, $R=[-\pi,\pi]\times[-\pi,\pi]$

#39. Use WolframAlpha to compute $\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx$ and $\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dx\,dy$.
Do your answers contradict Fubini's Theorem? Explain.