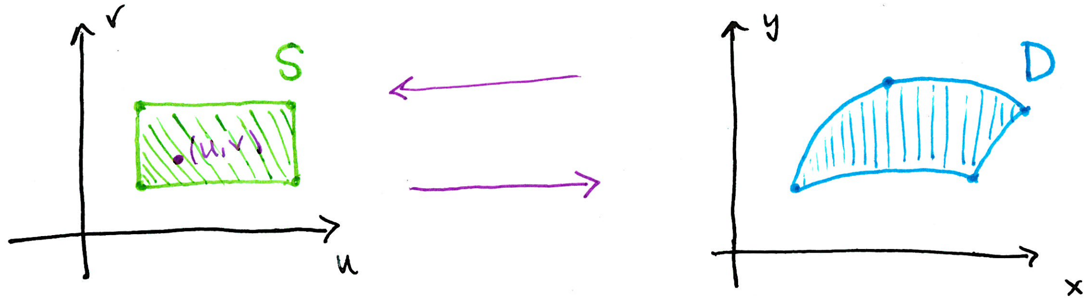
Section 15.10 - Change of Variables

★ Cylindrical and spherical coordinates are not the only coordinate systems. We can create lots of other coordinate systems.



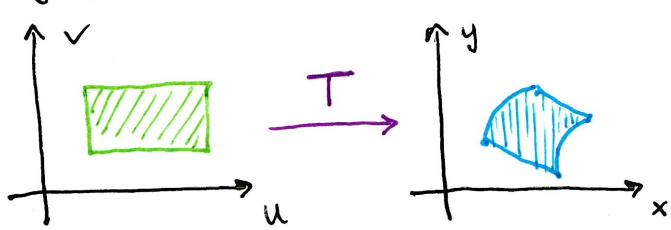
Transformation:

One-to-One Transformation:

C^1 Transformation:

Mapping Boundaries:

- Changing variables:



Goal:

★ Visit website to see demos

T: $\vec{r}(u,v) = \langle g(u,v), h(u,v) \rangle = \langle x, y \rangle$

- Change of variables in Double Integrals:

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$\boxed{\text{Example}}$ Show change of variables from Cartesian to polar in a double integral gives $dA = r\,dr\,d\theta$.

$\boxed{\text{Example}}$ Use the transformation defined by $x = u^2 - v^2$ and $y = 2uv$, to evaluate $\iint_R y\,dA$ where R is bounded by the x-axis and parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$, $y \geq 0$.

$\boxed{\text{Example}}$ Evaluate $\iint_R e^{\left(\frac{(x+y)}{(x-y)}\right)}\,dA$ where R is the trapezoid region with vertices $(0,-1)$ $(0,-2)$ $(2,0)$ $(1,0)$.

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MVC

- Change of Variables for Triple Integrals:

$T: \quad x = g(u,v,w) \qquad y = h(u,v,w) \qquad z = k(u,v,w)$

Jacobian of T: $\quad \dfrac{\partial(x,y,z)}{\partial(u,v,w)} =$

$$\iiint_R f(x,y,z)\,dV =$$

$\boxed{\text{Example}}$ Derive the formula for Triple integrals in spherical coordinates.

- Extra Examples:

\# 17 Evaluate $\iint_R x^2\,dA$, where R is the region bounded by $9x^2 + 4y^2 = 36$, use $x = 2u$, $y = 3v$.

\# 28 Let f be continuous on $[0,1]$ and let R be the triangle with vertices $(0,0)$ $(0,1)$ $(1,0)$. Show that $\iint_R f(x+y)\,dA = \int_0^1 u f(u)\,du$.