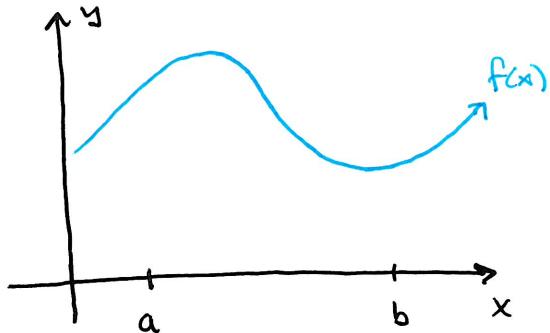


## Section 15.1 - Double Integrals over Rectangles

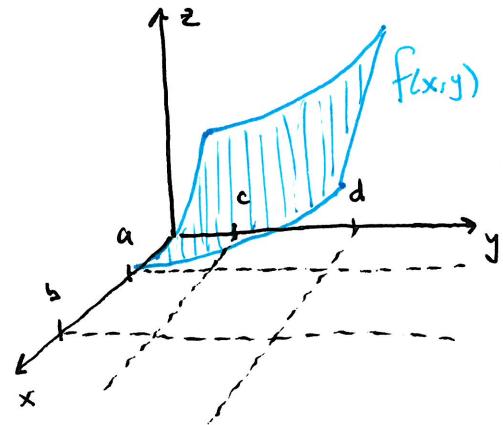
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- Review the Definite Integral:



\* Watch Riemann Sum Demo on Website

- Volumes and double Integrals:



Visual \* See Double Integral Demo on Website

- Volume of a rectangular cylinder:

- Approximate Volume under  $f(x,y)$ :

- Exact Volume under  $f(x,y)$ :

The double Integral of  $f$  over  $R = [a,b] \times [c,d]$  =

? Warning:

## Section 15.1 - Double Integrals over Rectangles

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Example

Estimate the volume of the solid that lies above the square  $R = [0, 2] \times [0, 2]$  and below  $z = 16 - x^2 - 2y^2$  by dividing  $R$  into 4 equal squares and choosing the upper right corner of each square for taking the height of the rectangular prism. Compare this approximation to the midpoint approximation.

- Average Value of  $f(x)$ :  
on  $[a, b]$
- Average Value of  $f(x, y)$ :  
on  $R$
- Properties of Double Integrals:

(1)

(2)

(3)

## Section 15.1 - Double Integrals over Rectangles

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### • Extra Examples

- #7 Let  $V$  be the volume of the solid under  $f(x,y) = \sqrt{52 - x^2 - y^2}$  and above the rectangle given by  $2 \leq x \leq 4, 2 \leq y \leq 6$ . Use  $x=3, y=4$  to divide  $R$  into subrectangles. Without computing the Riemann sums with the lower left corner ( $L$ ) and the upper right corner ( $R$ ) arrange  $V, L, R$  in increasing order.

- #12 Evaluate the double integral by identifying it as the volume of a solid,

$$\iint_R (5-x) dA \text{ where } R = [0, 5] \times [0, 3].$$

- #17 If  $f$  is a constant function  $f(x,y) = K$  and  $R = [a,b] \times [c,d]$

Show that  $\iint_R K dA = K(b-a)(d-c)$