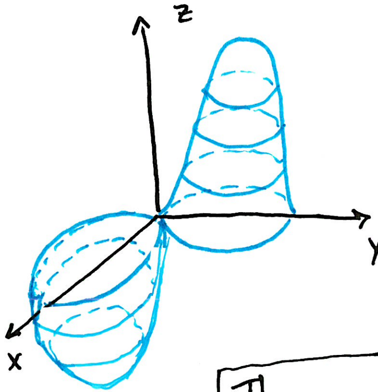
# Section 14.7 - Max and Min Values

MVC




$(a,b)$ a point in the domain of $f(x,y)$ is a:

- local min if
- local max if
- Absolute min if
- Absolute max if

**Theorem** If $f$ has a local max/min at $(a,b)$ and $f_x(a,b)$ and $f_y(a,b)$ exist then $\nabla f(a,b) = \vec{0}$.

Proof: $(a,b)$ local min/max of $f(x,y)$ is still a local min/max of $f(a,y)$ and $f(x,b)$ which are functions of one-variable. Thus

$$\frac{d}{dx} f(x,b)\Big|_{x=a} = f_x(a,b) = 0 \text{ also } f_y(a,b) = 0. \blacksquare$$

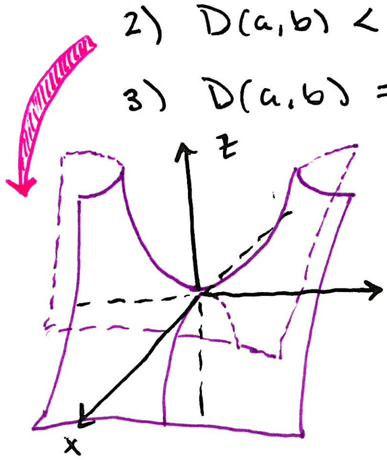
$(a,b)$ is a critical point of $f$ if

2 point can be a

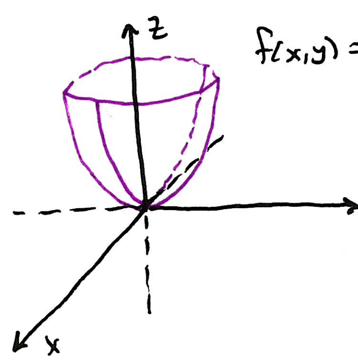
- Second Derivative Test: 2nd partials of $f$ continous on disk containing $(a,b)$ where $\nabla f(a,b) = \vec{0}$. Define:

$D(a,b) =$

1) $D(a,b) > 0$ then
   - Local max if
   - Local min if
2) $D(a,b) < 0$ then
3) $D(a,b) = 0$ then




$f(x,y) = y^2 - x^2$




$f(x,y) = x^2 + y^2$

Example Find the local max/min values and any saddle points of
$$f(x,y) = x^4 + y^4 - 4xy + 1$$

Example Find the shortest distance from the point $(1,0,-2)$ to the plane: $x + 2y + z = 4$.

- Extreme Value Theorem (EVT): Existence Theorem!

(1) $y = f(x)$ continous on a closed interval $[a,b]$

(2) $z = f(x,y)$ continuous on a

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- **Critical Point Theorem:** for functions on a closed bounded set

The absolute max/min value of:

(1) $y = f(x)$ occurs at either a

(2) $z = f(x,y)$ occurs at either a

**Example** Find the absolute max/min value of $f(x,y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x,y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.

**Example** Same function on the triangle whose vertices are $(0,0), (1,0), (0,1)$