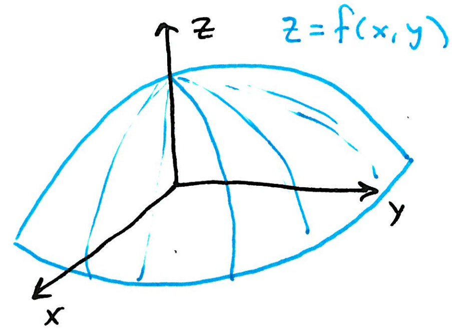
Section 14.6 - Directional Derivatives & the Gradient

- Directional Derivative:



at (x_0, y_0) in the direction of $\vec{u} = \langle a, b \rangle$ (unit vector) of $f(x,y)$ is

Theorem If f is a differentiable function of x and y, then f has a directional derivative for any direction $\vec{u} = \langle a, b \rangle$ (unit vector) and

Example 2 Find the directional derivative if $f(x,y) = x^3 - 3xy + 4y^2$ and $\vec{u}$ is the unit vector given by $\theta = \pi/6$. Find $D_{\vec{u}} f(1,2)$.

Note:

- Gradient of f:

Example $f(x,y,z) = y \ln(x^2 + z)$ find ∇f and $D_{\vec{u}} f$ in the direction of $\vec{v} = \langle 1, -1, 1 \rangle$ at $(0, 5, 1)$.

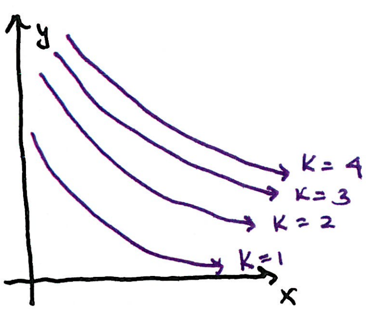
- Question: How would you maximize the directional derivative?
  (that is find the max of $D_{\vec{u}}f$ for a point on f)

Theorem If f is a differentiable function then the max value of the directional derivative $D_{\vec{u}} f(\vec{x})$ is:

and it occurs in the direction of:

Example 7 Suppose that the temp at a point (x,y,z) in space is given by $T(x,y,z) = 80(1+x^2+2y^2+3z^2)^{-1}$ °C where x,y,z are in meters. In what direction is the temp increasing fastest at $(1,1,-2)$ and what is the max rate of increase?

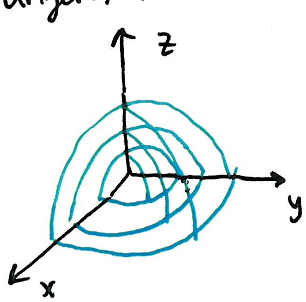
- Level Curves: $f(x,y) = k$ for $z = f(x,y)$



- Draw the gradient vectors on the level curves

★ Do you see a relation between the gradient and another vector?

- Tangent Plane to a Level Surface: $f(x,y,z) = k$ for $z = f(x,y,z)$



Tangent Plane to $f(x,y,z) = k$ at (x_0, y_0, z_0):

Section 14.6 - Directional Derivatives & The Gradient

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★ Watch: youtube.com/watch?v=NuNCIRnXWcE

- Review:

① $D_{\vec{u}} f(x,y) =$ ________ $\vec{u}$ must be ________

② $\nabla f =$ ________

③ Max value of $D_{\vec{u}} f(x,y)$ is ________

④ Direction of max value of $D_{\vec{u}} f(x,y)$ is ________

⑤ ∇f ___ tangent vectors on level curves (surfaces)

⑥ ∇f is the ________ for the tangent plane to $f(x,y,z) = k$

⑦ ∇f is the ________ of the normal line

⑧ On a level curve graph, ∇f points in the direction of ________

⑨ ∇f makes a ________ with the level curves

Example 8 Find the equations of the tangent plane and normal line at $(-2, 1, -3)$ to $x^2/4 + y^2 + z^2/9 = 3$.

#39 Second Directional Derivative:

$$D_{\vec{u}}^2 f(x,y) = D_{\vec{u}}\left(D_{\vec{u}} f(x,y)\right)$$

Find $D_{\vec{u}}^2 f(x,y)$ if $f(x,y) = x^3 + 5x^2y + y^3$ and $\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

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- Extra Examples:

#40 (a) If $\vec{u} = \langle a, b \rangle$ is a unit vector and f has continuous 2nd partials
Show that $D_{\vec{u}}^2 f = f_{xx} a^2 + 2 f_{xy} ab + f_{yy} b^2$

55 Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to $x + y = z$?

61 Show that the sum of the x, y, z intercepts of any tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{d}$ is a constant.

#67 Suppose $D_{\vec{u}} f(x,y)$ and $D_{\vec{v}} f(x,y)$ are know for two non-parallel vectors $\vec{u}, \vec{v}$.
Is it possible to find $\nabla f(x,y)$? If so how?