Section 14.5 - Chain Rule

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Recall: For $y=f(x)$ and $x=g(t)$, $y=f(g(t))$, where both f and g are differentiable then:

- Chain Rule (Case 1): $z=f(x,y)$ differentiable with $x=g(t)$ and $y=h(t)$ both differentiable then:

- Chain Rule (Case 2): $z=f(x,y)$ differentiable with $x=g(s,t)$ and $y=h(s,t)$ both differentiable then:

Example 2 The pressure P (in kPa), volume V (in L), temp T (in K) of a mole of an ideal gas are related by $PV=8.31T$. Find the rate at which the pressure is changing when the temp is 300 K increasing at 0.1 K/sec and the volume is 100 L increasing at 0.2 L/sec.

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Example 5 If $u = x^4y + y^2z^3$ where $x = rse^t$, $y = rs^2e^{-t}$ and $z = r^2s\sin t$ then find $\partial u/\partial s$ when $r=2$, $s=1$, $t=0$.

- **Implicit Differentiation:** suppose $F(x,y) = 0$ defines y implicitly as a differentiable function of x, $f(x) = y$ with $F(x, f(x)) = 0$ then:

Implicit Function Theorem $F(x, y) = 0$, F differentiable, $F_y \neq 0$ then $dy/dx =$

If $F(x,y,z) = 0$, $z = f(x,y)$ implicitly defined, F differentiable, $F_z \neq 0$

then: $\partial z/\partial x =$ $\partial z/\partial y =$

Example 8 Find dy/dx if $x^3 + y^3 = 6xy$.

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- Extra Examples

33 Find $\partial z/\partial x$ and $\partial z/\partial y$ where $e^z = xyz$.

39 The length l, width w, and height h of a box change with time. When $l = 1m$, $w = h = 2m$ and l and w are increasing at $2m/s$ while h is decreasing at $3m/s$. Find the rate of change in

(a) volume

(b) surface area

(c) length of diagonal

45 If $z = f(x,y)$ and $x = r\cos\theta$, $y = r\sin\theta$ show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$