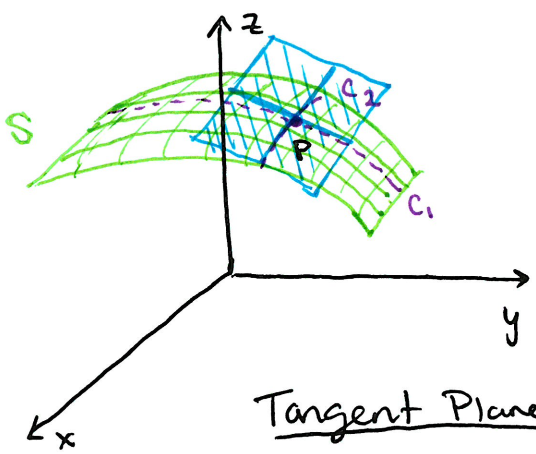
Section 14.4 - Tangent Planes



- S given by $z = f(x,y)$ with continuous first partials
- P a point (x_0, y_0, z_0) on S
- $C_1 = f(x_0, y)$ and $C_2 = f(x, y_0)$

Tangent Plane to S at P:

$\boxed{\text{Example 1}}$ Find the tangent plane to $z = 2x^2 + y^2$ at $(1, 1, 3)$.

- Linear Approximation:

- $f(x,y)$ Differentiable:

$\boxed{\text{Theorem}}$ If f_x and f_y exist near (a,b) and are continuous at (a,b) then f is differentiable at (a,b).

Proof: $\Delta z = f(a+\Delta x, b+\Delta y) - f(a,b)$ let $a' = a + \Delta x$ and $b' = b + \Delta y$

$= [\underbrace{f(a', b') - f(a, b')}_{\text{function of } x}] + [\underbrace{f(a, b') - f(a,b)}_{\text{function of } y}]$

by MVT $= f_x(x_a, b')\Delta x + f_y(a, y_b)\Delta y$ where $x_a \in (a, a+\Delta x)$ $y_b \in (b, b+\Delta y)$

$= f_x(a,b)\Delta x + [\underbrace{f_x(x_a, b') - f_x(a,b)}_{\text{continuous} \to (a,b)}]\Delta x + f_y(a,b)\Delta y + [\underbrace{f_y(a, y_b) - f_y(a,b)}_{\text{continuous} \to (a,b)}]\Delta y$

$= f_x(a,b)\Delta x + \varepsilon_1 \Delta x + f_y(a,b)\Delta y + \varepsilon_2 \Delta y$ where $\varepsilon_1, \varepsilon_2 \to 0$ as $\Delta x, \Delta y \to 0$ ■

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$\boxed{\text{Example 2}}$ Show $f(x,y) = xe^{xy}$ is differentiable at $(1,0)$ and find its Linearization at $(1,0)$ to approximate $f(1.1, -0.1)$.

• Differentials:

Two variables

One variable

$\boxed{\text{Example 5}}$ The base radius and height of a right circular Cone are measured as 10cm and 25cm, with a Possible error of 0.1cm in each. Use differentials to estimate the max error in Calculating the volume of the cone, then check by computing two volumes.

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- Extra Examples

#31 If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$
Compare Δz and dz.

#38 The pressure, volume, and temp of a mole of an ideal gas are related by the equation $PV = 8.31T$ where P is measured in KPa, V in L, T in K. Use differentials to find the approx. change in pressure if V is increased from 12L to 12.3L and the temp decreases from 310 K to 305 K.

#42 $\vec{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2 \rangle$ and $\vec{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1 \rangle$ lie on S and contain $(2,1,3)$. Find the tangent plane to S at $(2,1,3)$.