Section 14.3 - Partial Derivatives

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- Recall: Definition of the derivative of $y=f(x)$ at $x=a$

- Now: For a function $z=f(x,y)$ only vary x, fix y as a constant $y=b$

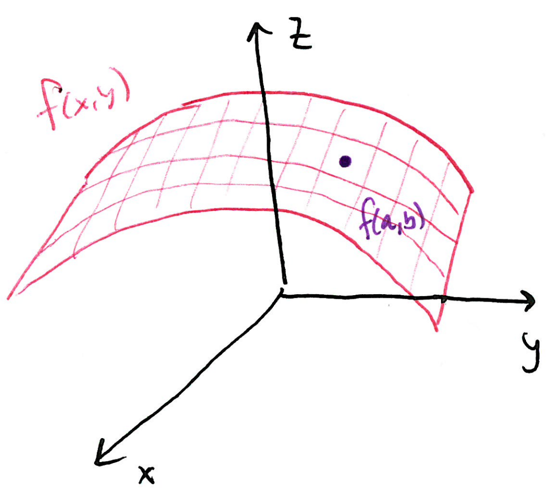
- Notation for Partial Derivatives: $z=f(x,y)$

$f_x(x,y) =$

$f_y(x,y) =$

Example $f(x,y) = x^2\sin(y) + x\ln(x+y^2)$, Find $f_x(2,0)$ and $f_y(2,0)$

- Interpretation:



Example 4 $x^3+y^3+z^3+6xyz=1$
Find $\partial z/\partial x$ and $\partial z/\partial y$.

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- Higher Order Derivatives:

Notation:

$\boxed{\text{Example 6}}$ Find the Second Partial derivatives of $f(x,y) = x^3 + x^2y^3 - 2y^2$

$\boxed{\text{Clairaut's Theorem}}$ f defined on D containing (a,b). If f_{xy} and f_{yx} are continuous on D then:

$\boxed{\text{Example}}$ Show $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ fails Clairaut's Theorem at $(0,0)$. Why?

- Partial Differential Equations:

Example: Laplace Equation

Solutions are called Harmonic Functions

↳ used in Heat Conduction, fluid flow, electric Potential

$\boxed{\text{Example 8}}$ Show $f(x,y) = e^x \sin y$ is a solution of the Laplace Equation.

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- Extra Examples:

#9 See page 936 label graphs a, b, c as f, f_x, f_y give reasons.

#71 $f(x,y,z) = xy^2z^3 + \arcsin(x\sqrt{z})$ find f_{xzy} (Hint: Which order is easier?)

#83 Total resistance R produced by 3 conductors with resistance R_1, R_2, R_3 and connected in a parallel electrical circuit is $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
Find $\partial R / \partial R_1$

#88 The gas law for a fixed mass m of an ideal gas at absolute temp T, pressure P, and volume V is $PV = mRT$ where R is the gas constant. Show

$$\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1$$

#93 Is there a function f with $f_x(x,y) = x + 4y$ and $f_y(x,y) = 3x - y$?