Section 13.4 - Motion in Space

We can use vector functions to describe particle motion:

- Position:
- Velocity:
- acceleration:
- Speed:

- **Particle Motion is Cool:** youtube.com/watch?v=FG_11oacWbQ

Each water droplet can be thought of as a particle, each only given acceleration due to gravity.

Example 3 A moving particle starts at an initial position $\vec{r}(0) = \langle 0,0,0 \rangle$ with initial velocity $\vec{v}(0) = \langle 1,-1,1 \rangle$. It's acceleration is $\vec{a}(t) = \langle 4t, 6t, 1 \rangle$. Find its velocity and position at time t.

- **Newton's Second Law of Motion:**

$\vec{F}$ Force acting on an object of mass m produces acceleration $\vec{a}$

Example 5 A projectile is fired with an angle of elevation α and initial velocity $\vec{v}_0$. Assume air resistance is negligible and the only external force is gravity. Find $\vec{r}(t)$ and α that maximizes the horizontal range.

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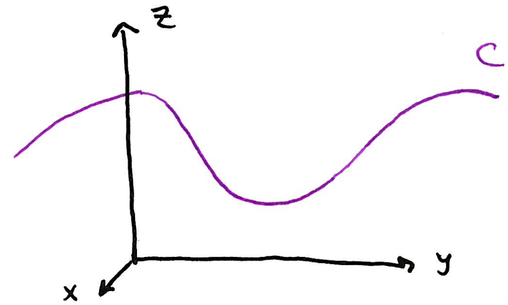
- Tangential & Normal Comporets of acceleration:



Tangential Acceleration:

Normal Acceleration:

$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\vec{v}}{v}$ where $v = |\vec{v}|$ so $\vec{v} = v\vec{T}$ [velocity in terms of speed & unit tangent]

Diffentiate both sides wrt t:

Example #36

(a) If a particle mores along a straight line, what can be said about its acceleration vector?

(b) If a particle mores with constant speed along a curve, what can be said about its acceleration vector?

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- Extra Examples

#22 Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

#25 A ball is thrown at 45° to the ground. If the ball lands 90m away, what was the initial speed?

#45 The position of a spaceship is: $\vec{r}(t) = \langle (3+t), (2+\ln t), (7 - \frac{4}{t^2+1}) \rangle$ and the coordinates of a space station are $(6,4,9)$. The Captain wants the space ship lined up with the space station so it can coast in with engines off. When should he turn off the engines?