Section 13.2 - Derivatives & Integrals of Vector Functions

MVC

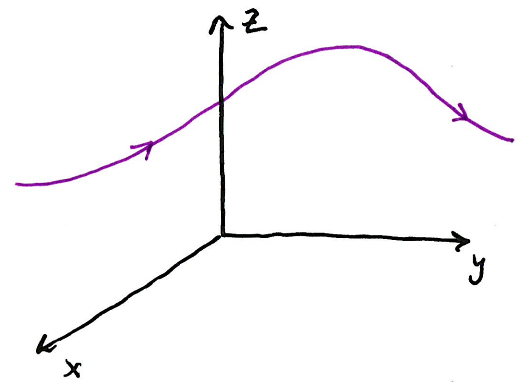
Recall: The derivative of a function $y=f(x)$ at a point $x=a$ represents the slope of the line tangent to $f(x)$ at $x=a$.

- First approx. slope between two points $(a, f(a))$ and $(a+h, f(a+h))$

$$\frac{f(a+h)-f(a)}{h}$$

- Limit as $h \to 0$ gave the slope at one point $x=a$ i.e. slope of tangent line at $x=a$.

- Tangent Vector (Derivative of $\vec{r}(t)$) $\vec{r}\,'(t)$:



$\boxed{\text{Theorem}}$ If $\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$ and f, g, h are differentiable, then

$$\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t)\rangle$$

Proof: $$\vec{r}\,'(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \lim_{\Delta t \to 0} \left\langle \frac{f(t+\Delta t)-f(t)}{\Delta t}, \frac{g(t+\Delta t)-g(t)}{\Delta t}, \frac{h(t+\Delta t)-h(t)}{\Delta t} \right\rangle$$

$$= \langle f'(t), g'(t), h'(t)\rangle$$ since f, g, h are differentiable. ∎

$\boxed{\text{Example 1}}$ (a) Find the derivative of $\vec{r}(t) = \langle (1+t^3), te^{-t}, \sin 2t\rangle$

(b) Find the unit tangent vector to $\vec{r}(t)$ when $t=0$.

$\boxed{\text{Example 3}}$ Find parametric equations for the tangent line to the helix:

$x = 2\cos t \quad y = \sin t \quad z = t \quad$ at $\quad (0, 1, \pi/2)$

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- **Differentiation Rules**: $\vec{u}, \vec{v}$ differentiable, c a scalar, f differentiable

1. $\frac{d}{dt}(\vec{u}(t)+\vec{v}(t)) =$
2. $\frac{d}{dt}(c\vec{u}(t)) =$
3. $\frac{d}{dt}(f(t)\,\vec{u}(t)) =$
4. $\frac{d}{dt}(\vec{u}(t)\cdot\vec{v}(t)) =$
5. $\frac{d}{dt}(\vec{u}(t)\times\vec{v}(t)) =$
6. $\frac{d}{dt}(\vec{u}(f(t))) =$

Example 4 Show that if $|\vec{r}(t)| = c$ then $\vec{r}\,'(t)$ is orthogonal to $\vec{r}(t)$.

- **Definite Integral**: $\int_a^b \vec{r}(t)\,dt =$

- **Indefinite Integral**: $\int \vec{r}(t)\,dt =$

★ FTC:

Example 5 If $\vec{r}(t) = \langle 2\cos t, \sin t, 2t\rangle$ find $\int \vec{r}(t)\,dt$ and $\int_0^{\pi/2} \vec{r}(t)\,dt$?

- Question: What does $\int_a^b \vec{r}(t)\,dt$ represent?

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- Extra Examples

#27 Find a vector equation for the tangent line to the curve of intersection of:

$$x^2+y^2=25 \quad \text{and} \quad y^2+z^2=20 \quad \text{at} \quad (3,4,2)$$

#33 $\vec{r}_1(t)=\langle t,t^2,t^3\rangle$ and $\vec{r}_2=\langle \sin t, \sin 2t, t\rangle$ both intersect the origin,
Find their angle of intersection.

#40 $\int\left(te^{2t}\vec{i}+\frac{t}{1-t}\vec{j}+\frac{1}{\sqrt{1-t^2}}\vec{k}\right)dt$

#53. If $\vec{r}(t)\neq\vec{0}$ show that $\frac{d}{dt}|\vec{r}(t)|=\frac{1}{|\vec{r}(t)|}\vec{r}(t)\cdot\vec{r}\,'(t)$. Hint: $|\vec{r}(t)|^2=\vec{r}(t)\cdot\vec{r}(t)$