Section 12.3 - The Dot Product

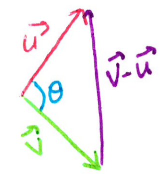
MVC

Q: Is it possibly to "multiply" two vectors with the result being meaningful?

- **Dot Product** (Scalar Product): $\vec{a} = \langle a_1, a_2, a_3 \rangle$ $\vec{b} = \langle b_1, b_2, b_3 \rangle$

  $\vec{a} \cdot \vec{b} =$

$\boxed{\text{Theorem}}$ If θ is the angle between $\vec{v}$ and $\vec{u}$ then $\vec{v} \cdot \vec{u} = |\vec{v}||\vec{u}| \cos\theta$



Proof: Apply Law of Cosines to the triangle formed by $\vec{u}$ and $\vec{v}$

$$|\vec{v}-\vec{u}|^2 = |\vec{v}|^2 + |\vec{u}|^2 - 2|\vec{v}||\vec{u}| \cos\theta$$

in V^2: $(v_1-u_1)^2 + (v_2-u_2)^2 = v_1^2 + v_2^2 + u_1^2 + u_2^2 - 2|\vec{u}||\vec{v}|\cos\theta$

$$-2v_1u_1 - 2v_2u_2 = -2|\vec{u}||\vec{v}|\cos\theta$$

Meaningful quantity why we define the Dot product

$$\underbrace{v_1u_1 + v_2u_2} = |\vec{u}||\vec{v}|\cos\theta$$

$$\vec{v} \cdot \vec{u} = |\vec{u}||\vec{v}|\cos\theta \quad \blacksquare$$

- Properties of Dot Product:

1. $\vec{a} \cdot \vec{a} =$
2. $\vec{a} \cdot \vec{b} =$
3. $\vec{a} \cdot (\vec{b} + \vec{c}) =$
4. $(c\vec{a}) \cdot \vec{b} =$
5. $\vec{0} \cdot \vec{a} =$

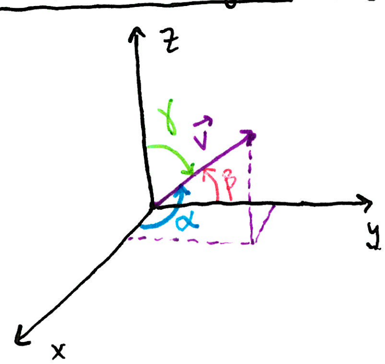
$\boxed{\text{Corollary}}$ $\cos\theta = \dfrac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|}$

$\boxed{\text{Corollary}}$ $\vec{v}$ and $\vec{u}$ are orthogonal (perpendicular) if and only if $\vec{v} \cdot \vec{u} = 0$ with $\vec{v}, \vec{u} \neq \vec{0}$.

Proof:

$\boxed{\text{Example 3}}$ Find the angle between $\vec{a} = \langle 2, 2, -1 \rangle$ and $\vec{b} = \langle 5, -3, 2 \rangle$

- Directional Angles: The angles $\vec{v}$ makes with the positive x, y, z axes



$\cos\alpha =$ $\qquad$ $\cos\beta =$ $\qquad$ $\cos\gamma =$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

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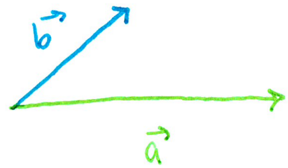
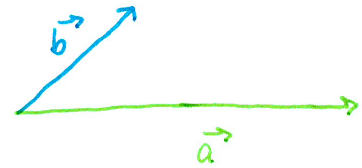
- **Projections:**

Scalar Projection of $\vec{b}$ onto $\vec{a}$



$\text{Comp}_{\vec{a}}\,\vec{b} =$

Vector Projection of $\vec{b}$ onto $\vec{a}$

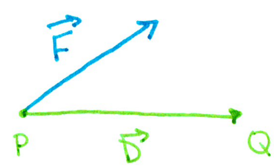


$\text{Proj}_{\vec{a}}\,\vec{b} =$

Example 6 Find the scalar and vector projections of $\vec{u} = \langle 1, 1, 2 \rangle$ onto $\vec{v} = \langle -2, 3, 1 \rangle$

- **Applications:** Work – force constant in direction of displacement $W = F \cdot |\vec{D}|$

★ Constant Force not in direction of $\vec{D}$ given by $\vec{F}$



Work $W =$

Example A wagon is pulled a distance of 100m along a horizontal path by a constant force of 70N. The handle is held at some angle above the horizon. If the work done was about 5734 J find the angle of the handle above the horizon.

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- Extra Examples:

① If $\vec{a} = \langle 1, 2, 3 \rangle$ find $\vec{b}$ so that $\text{comp}_{\vec{a}}\, \vec{b} = 3$.

#48. Suppose $\vec{a}$ and $\vec{b}$ are nonzero vectors. When is $\text{comp}_{\vec{a}}\, \vec{b} = \text{comp}_{\vec{b}}\, \vec{a}$?

55. Find the angle between a diagonal of a cube and one of its edges.

61. Use Theorem 3 to prove Cauchy - Schwarz Inequality: $|\vec{a} \cdot \vec{b}| \leq |\vec{a}|\,|\vec{b}|$