

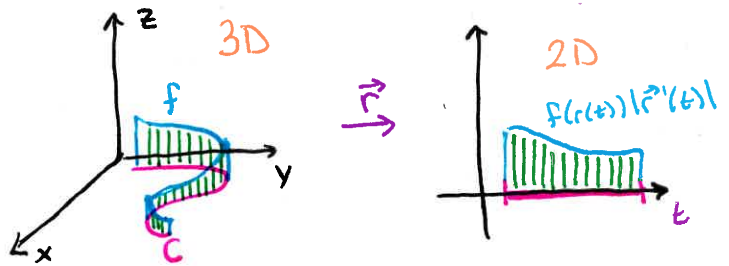
Where we've been (16.1-16.7) and where we're going (16.8-16.9)

Line Integrals:

- Scalar function

$$\int_C f \, ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| \, dt$$

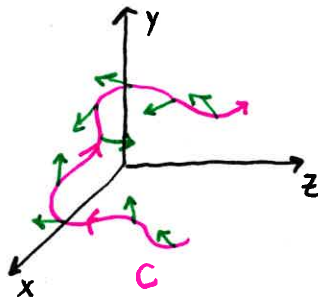
* Accumulation of f over C
or Area of f over C



- Vector Function

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt$$

* Amount of \vec{F} going
in the direction
of C



Theorems

① \vec{F} Conservative | Line Integrals with vector functions

FTC $\int_C \nabla f \cdot d\vec{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$ C any curve

② \vec{F} Not Conservative | Line Integrals with vector function (2 vars)

Green's

$$\int_C \vec{F} \cdot d\vec{r} = \int_C P \, dx + Q \, dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

C positive orientation, simple, closed

Where we've been and where we're going

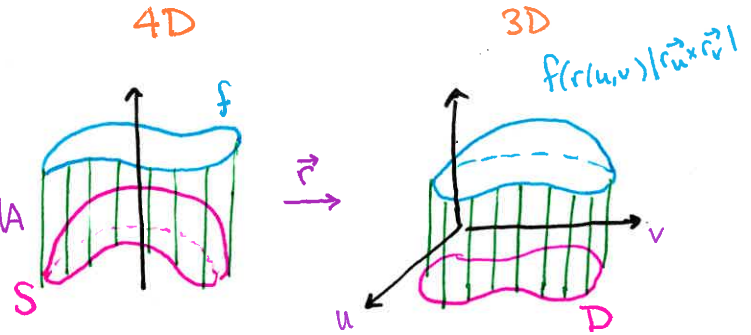
MVC

Surface Integrals:

• Scalar functions

$$\iint_S f dS = \iint_D f(r(u,v)) |\vec{r}_u \times \vec{r}_v| dA$$

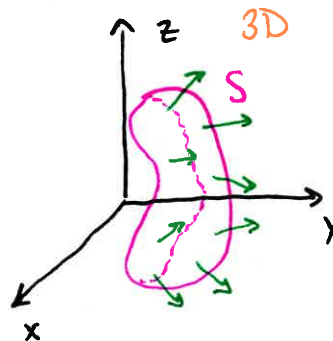
★ Accumulation of f over S or
Volume of f over S



• Vector Functions

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(r(u,v)) \cdot (\vec{r}_u \times \vec{r}_v) dA$$

★ Amount of \vec{F} going
through S



Theorems

③ \vec{F} not Conservative | Line Integrals with Vector Functions (3 vars)

Stoke's ~ Green's theorem for functions of 3 variables

④ \vec{F} Not Conservative | Surface Integrals with Vector Functions

Divergence