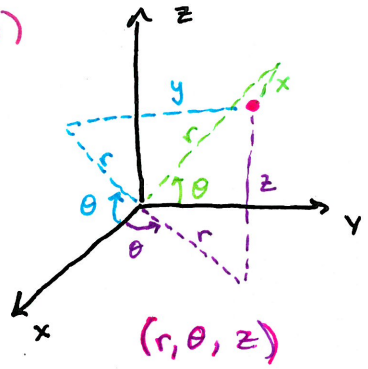
Section 15.8 - Cylindrical Coordinates

MVC

★ 3D equivalent to Polar Coordinates : (r, θ, z)



In general: $\boxed{x = r\cos\theta, \quad y = r\sin\theta, \quad z = z}$

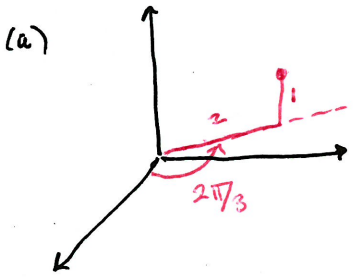
or $x = r\cos\theta, \quad z = r\sin\theta, \quad y = y$

or $y = r\cos\theta, \quad z = r\sin\theta, \quad x = x$

$\boxed{r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}}$

Example (a) Plot $(2, 2\pi/3, 1)$ find the Cartesian Coords
(b) Find Cylindrical Coords for $(3, -3, -7)$

(a)



$x = 2\cos\left(\frac{2\pi}{3}\right) = -1$

$y = 2\sin\left(\frac{2\pi}{3}\right) = \sqrt{3}$

$z = 1$

$\boxed{(-1, \sqrt{3}, 1)}$

(b) $r^2 = (3)^2 + (-3)^2 = 18 \Rightarrow r = 3\sqrt{2}$

$\tan\theta = \frac{-3}{3} = -1 \quad \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

Since $(3, -3, -7)$ is in the 4th quadrant

$\theta = 7\pi/4$ or $-\pi/4$

$\boxed{(3\sqrt{2}, 7\pi/4, -7)}$

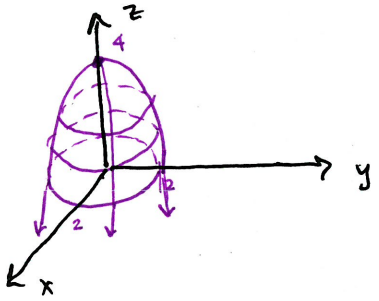
Example Identify and sketch the surface $z = 4 - r^2$.

$r^2 = x^2 + y^2$

$z = 4 - (x^2 + y^2)$

⇓

Paraboloid reflected over xy-plane up 4 units



Example A solid E lies within $x^2 + y^2 \le 1$, below $z = 4$, above $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

$E = \{(r, \theta, z) \mid 0 \le \theta \le 2\pi, \; 0 \le r \le 1, \; 1 - r^2 \le z \le 4\}$

$\rho(x, y, z) = K\sqrt{x^2 + y^2} = Kr$

$$m = \iiint_E \rho(x, y, z)\, dV = \int_0^{2\pi}\int_0^1\int_{1-r^2}^4 K \cdot r^2 \, dz\, dr\, d\theta = 2\pi K \int_0^1 4r^2 - r^2 + r^4 \, dr$$

$$= \boxed{2\pi K\left(1 + \frac{1}{5}\right)}$$

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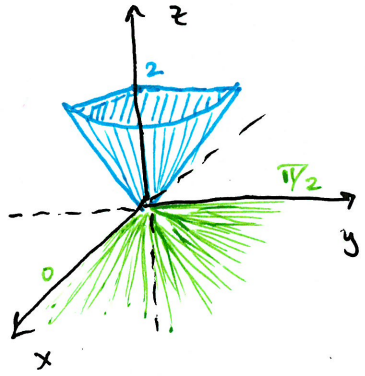
- Extra Examples:

#12. Sketch the solid described by $0 \le \theta \le \pi/2$, $r \le z \le 2$



1/4 of a solid cone

#17. Evaluate $\iiint_E \sqrt{x^2+y^2}\, dV$, where E is the region that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-5$ and $z=4$.

$$= \int_0^{2\pi}\int_0^4\int_{-5}^{4} r\, dz\, r\, dr\, d\theta = 2\pi(9)\int_0^4 r^2\, dr = \boxed{6\pi(4)^3}$$

#21. Evaluate $\iiint_E x^2\, dV$, where E is the solid that lies within the cylinder $x^2+y^2=1$, above $z=0$ and below the cone $z^2 = 4x^2+4y^2$.

$z^2 = 4r^2$

$z = 2r \sin\theta \quad z \ge 0$

$$= \int_0^{2\pi}\int_0^1\int_0^{2r} x^2\, dz\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 2r^4\cos^2\theta\, dr\, d\theta$$

$$= \left(\frac{2}{5}\right)\cdot\frac{1}{2}\int_0^{2\pi} 1+\cos(2\theta)\, d\theta = \frac{1}{5}\left(2\pi + \sin(4\pi)^0\right) = \boxed{\frac{2\pi}{5}}$$