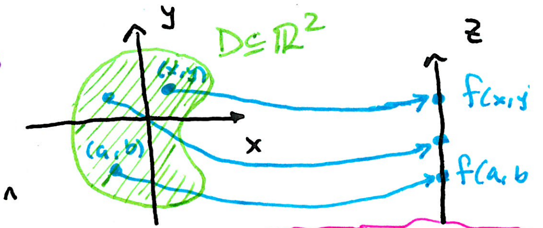
Section 14.1 - Functions of Several Variables

MVC

- **A function of two variables:** a rule that assigns to each pair $(x,y) \in D \subseteq \mathbb{R}^2$ a unique value $z = f(x,y) \in \mathbb{R}$



D is the domain of $z = f(x,y)$

Example 1 Evaluate $f(3,2)$ and sketch the domain

(a) $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

$f(3,2) = \dfrac{\sqrt{3+2+1}}{3-1} = \boxed{\dfrac{\sqrt{6}}{2}}$

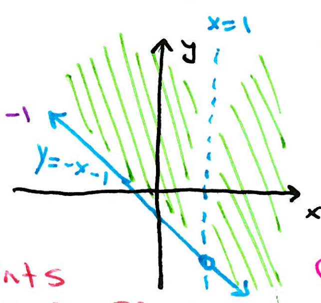
$D = \{(x,y) \mid y \geq -x-1,\ x \neq 1\}$

$\sqrt{n}$ defined only for $n \geq 0$

1) $x+y+1 \geq 0 \Rightarrow y \geq -x-1$

dividing by zero is undefined

2) $x - 1 \neq 0 \Rightarrow x \neq 1$



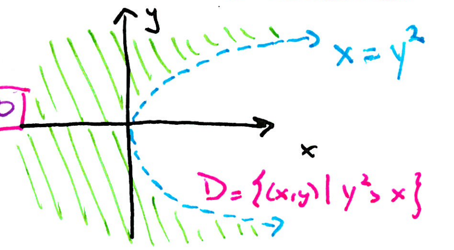
(b) $f(x,y) = x\ln(y^2 - x)$

$f(3,2) = 3\ln(1) = \boxed{0}$

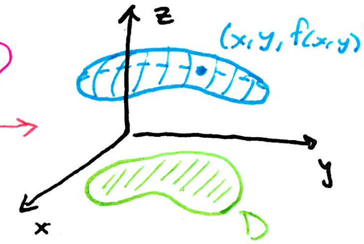
1) $y^2 - x > 0$

$y^2 > x$

ln(n) defined only for $n > 0$



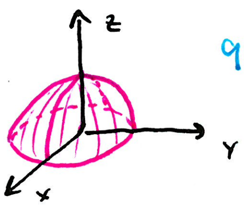
- **Graph of $f(x,y)$:** All points $(x,y,f(x,y)) \in \mathbb{R}^3$ with $(x,y) \in D \subseteq \mathbb{R}^2$.



Example 6 Sketch the graph of $g(x,y) = \sqrt{9-x^2-y^2}$

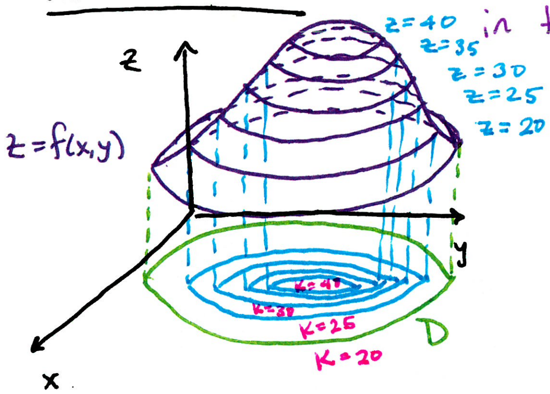
Note: $z = g(x,y)$ so $z^2 = 9 - x^2 - y^2$ with $z \geq 0$

$9 = x^2 + y^2 + z^2;\ z \geq 0 \rightarrow$ top sphere of radius 3



See pg. 906 for other Cool Surfaces

- **Level Curves:** Curves with equations $K = f(x,y)$ where K is a constant in the range of $z = f(x,y)$.



Why would we look at level curves?

1) 2D curves easier to graph than 3D surfaces

2) Easier to read info from level curves

3) Can be easier to visualize surface with curves

Examples:
- Topographic Maps - Pg. 907 figure 12

★ Watch: Augmented Reality Sandbox: youtube.com/watch?v=CE1B7tdGCw0

- Weather maps for Temp - Pg. 908 figure 13
  ↳ watch weather on News
  level curves called <u>isothermals</u>
- Medical Imaging

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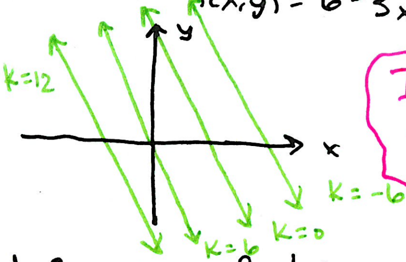
Example 10 Sketch the level curves of $f(x,y) = 6 - 3x - 2y$ for $k = -6, 0, 6, 12$

$k = 6 - 3x - 2y$

$y = \dfrac{-3x + 6 - k}{2}$

All lines with slope $-3/2$

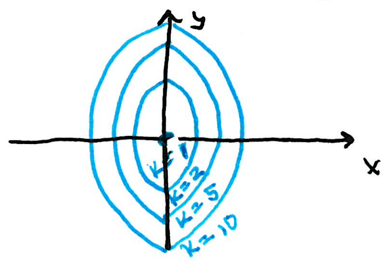


Imagine the lines above the page for $k>0$, on page for $k=0$ and below the page for $k<0$ ⇒ A plane surface

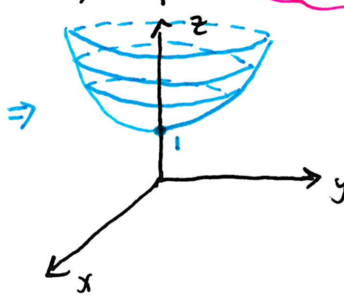
Example 12 Sketch some level curves of $h(x,y) = 4x^2 + y^2 + 1$

$k = 4x^2 + y^2 + 1 \rightarrow$ Ellipses

$\dfrac{x^2}{\frac{1}{4}(k-1)} + \dfrac{y^2}{k-1} = 1$ $\quad k=1$ point $(0,0)$



⇒



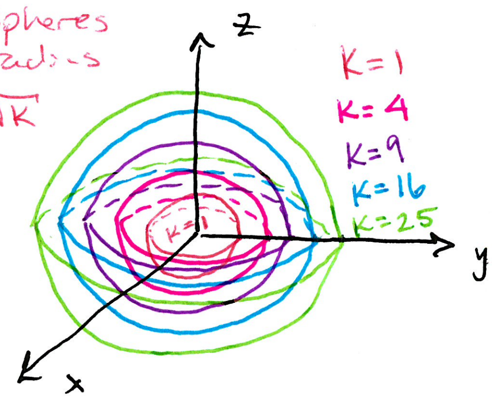
- <u>Functions of 3 variables</u>: rule assigning to each point $(x,y,z) \in D \subseteq \mathbb{R}^3$ a unique value $w = f(x,y,z) \in \mathbb{R}$.
- <u>Level Surface</u>:

★ We can't see in 4D but we can visualize how their 3D shadows change!

Think of 4D as a 3D movie watched all at once - you're outside of time

Example 15 Find the level surfaces of the function $f(x,y,z) = x^2 + y^2 + z^2$

Level Surfaces: $k = x^2 + y^2 + z^2 \leftarrow$ spheres of radius $\sqrt{k}$



- <u>Computer Visualizations</u>:

4D Sphere - Hypersphere: youtube.com/watch?v=BqfwPQvb7KA

4D Cube - Tesseract: youtube.com/watch?v=jGO1Z25Lw8s

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- Extra Examples

#32 Match the function with its graph: (Pg. 913)

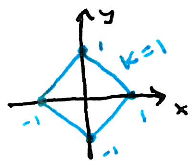
(a) $f(x,y) = |x| + |y|$

Level Curves: $k = |x| + |y|$

$k = x + y \quad k = x - y$

$k = -x + y \quad k = -x - y$

4 lines



VI

(b) $f(x,y) = |xy|$

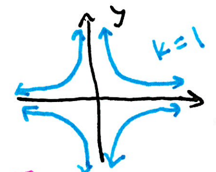
Level Curves: $k = |xy|$

$k = xy \quad 0 = xy$

$\Downarrow$

$k = -xy \quad x=0$ or $y=0$

2 reciprocal graphs

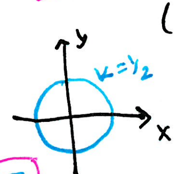


V

(c) $f(x,y) = \dfrac{1}{1+x^2+y^2}$

level curves: $k = \dfrac{1}{1+x^2+y^2}$

$x^2 + y^2 = \frac{1}{k} - 1$ circles



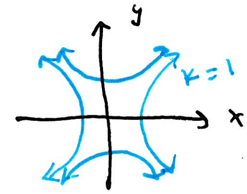
I

(d) $f(x,y) = (x^2 - y^2)^2$

$k = (x^2 - y^2)^2$

$x^2 - y^2 = \pm\sqrt{k}$

Hyperbola x-axis, y-axis



IV

(e) $f(x,y) = (x-y)^2$

$\pm\sqrt{k} = x - y$ two lines

II

(f) $f(x,y) = \sin(|x| + |y|)$

$k = \sin(|x|+|y|)$ or $z = \sin(|x|)\ y=0$

$z = \sin(|y|)\ x=0$

III

#36 Two contour maps are shown; one is a cone, one is a paraboloid. which is which and why?

Cone: $z^2 = x^2 + y^2$

level curves: $k^2 = x^2 + y^2$

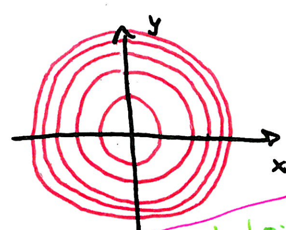
$\boxed{\text{radius } k \text{ increase proportional to } z}$

Paraboloid: $z = x^2 + y^2$

level curves: $(\sqrt{k})^2 = x^2 + y^2$

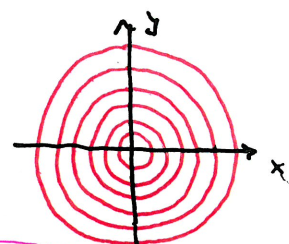
$\boxed{\text{Increase of } z \text{ proportional to } \sqrt{k}}$

I



Paraboloid

II



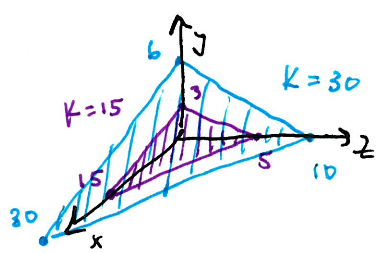
Cone

#65 Describe the level surfaces of $f(x,y,z) = x + 3y + 5z$

$k = x + 3y + 5z$

Equation of parallel planes all with normal vector:

$\langle 1, 3, 5 \rangle$



#69 Describe how g is obtained from f:

(a) $g(x,y) = f(x,y) + 2$ Shift f up two units on z-axis

(b) $g(x,y) = 2f(x,y)$ Stretch f by 2 along z-axis

(c) $g(x,y) = -f(x,y)$ flip f over the xy-plane

(d) $g(x,y) = 2 - f(x,y)$ flip f over the xy-plane up 2 on z-axis