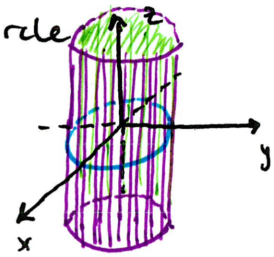
Section 12.6 – Cylinders & Quadric Surfaces

★ Conic Sections are 2D curves what about 3D surfaces?

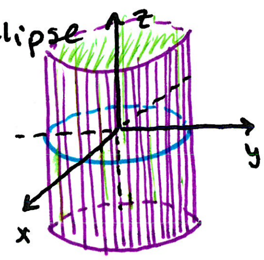
- **Cylinders**: surface of all lines (called rulings) parallel to one another, passing through a given plane curve perpendicular to the plane containing the plane curve.

$\boxed{\text{Example}}$ Sketch a cylinder for each conic section:

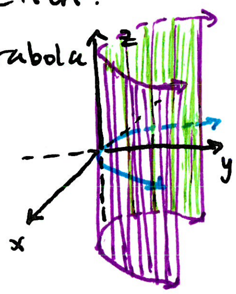
① Circle



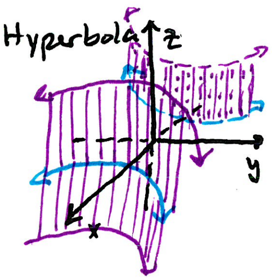
② Ellipse



③ Parabola



④ Hyperbola



- **Quadric Surfaces**: graph of a second-degree equation in 3-variables.

General Equation: $Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$

By translation and/or rotation all can be rewritten as:

$\boxed{Ax^2 + By^2 + Cz^2 + J = 0}$ or $\boxed{Ax^2 + By^2 + Iz = 0}$

4 possibilities — 2 possibilities

- Method of sketching: Layer traces – 2D graphs obtained by fixing 1 variable

$\boxed{\text{Example 3}}$ Use traces to sketch

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

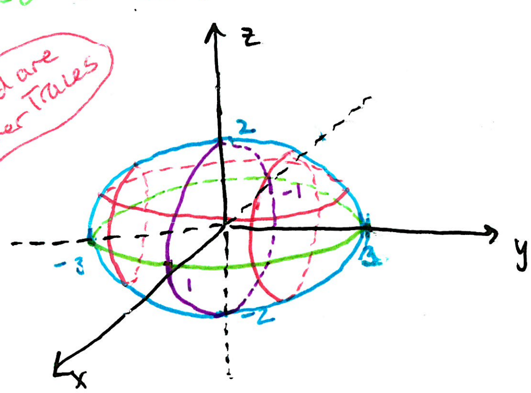
Traces:

$x=0$: Ellipse $\frac{y^2}{9} + \frac{z^2}{4} = 1$

$y=0$: Ellipse $x^2 + z^2/4 = 1$

$z=0$: Ellipse $x^2 + y^2/9 = 1$

In red are other Traces



$\boxed{\text{Example 5}}$ Sketch using traces

$$z = y^2 - x^2$$

$x=0$: parabola $z = y^2$

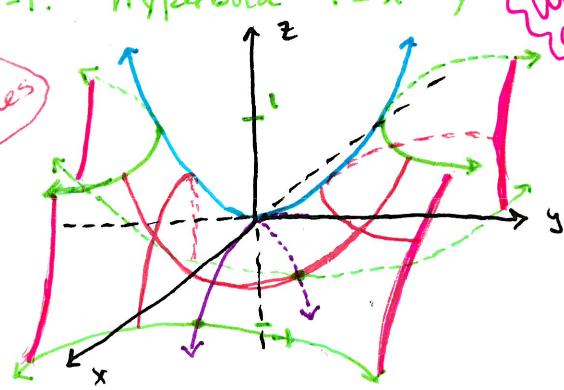
$y=0$: parabola $z = -x^2$

$z=1$: Hyperbola $1 = y^2 - x^2$

$z=-1$: Hyperbola $1 = x^2 - y^2$

looks like a saddle

In red are other Traces



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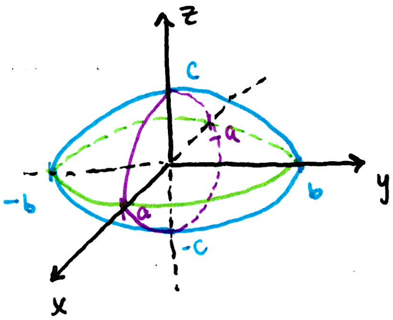
MVC

- 6 Types of Quadric Surfaces: meaningful ways to combine 3 conic sections

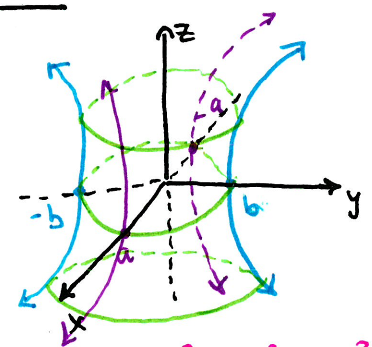
Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Traces:
Ellipse
Ellipse
Ellipse



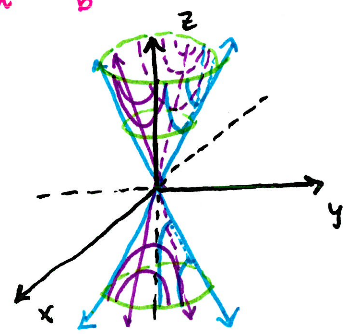
Hyperboloid of 1 Sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Traces:
Hyperbola about y
Hyperbola about x
Ellipse about z for all z



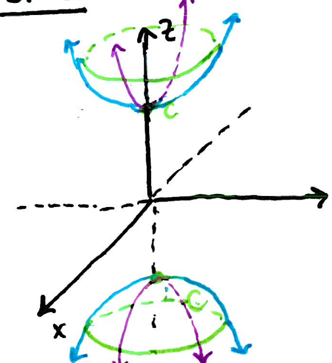
Cone: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

Traces:
Hyperbola about z
Hyperbola about z
Ellipse about z for all z



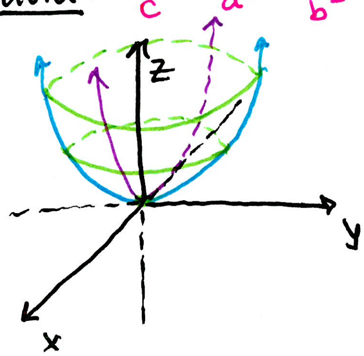
Hyperboloid of 2 Sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Traces:
Hyperbola about z
Hyperbola about z
Ellipse about z for $|z|\geq c$



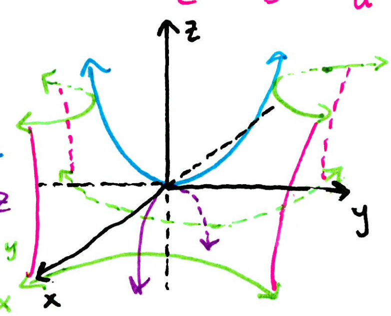
Elliptic Paraboloid: $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Traces:
Parabola about z
Parabola about z
Ellipse about z $z\geq 0$



Hyperbolic Paraboloid: $\frac{z}{c}=\frac{y^2}{b^2}-\frac{x^2}{a^2}$

Traces:
Parabola about z
Parabola about -z
$z>0$ Hyperbola about y
$z<0$ Hyperbola about x



Example 7 Identify & Sketch

$4x^2-y^2+2z^2+4=0$

$-x^2+\frac{y^2}{4}-\frac{z^2}{2}=1$

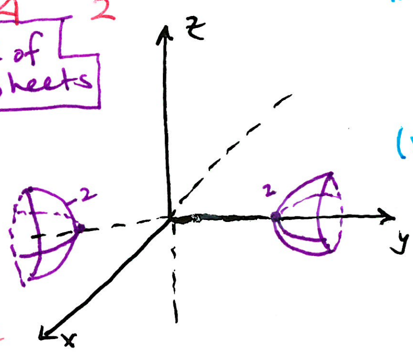
Hyperboloid of 2 Sheets

Traces:

1) $\frac{y^2}{2^2}-\frac{z^2}{2}=1$

2) $\frac{y^2}{2^2}-x^2=1$

3) $x^2+z^2/2=-1+K^2/4$ $|K|\geq 2$



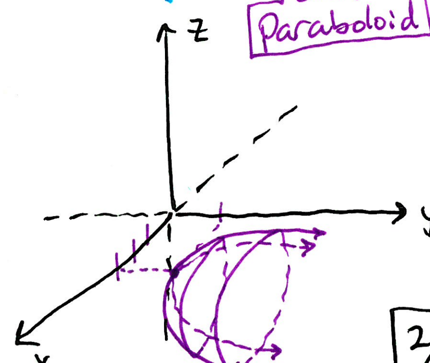
Example 8 Identify & Sketch

$x^2+2z^2-6x-y+10=0$

$(x-3)^2+2z^2=(y-1)$

Elliptic Paraboloid

Traces:
$(x-3)^2=y-1$
$2z^2=y-1$
$(x-3)^2+2z^2=K-1$
$K\geq 1$



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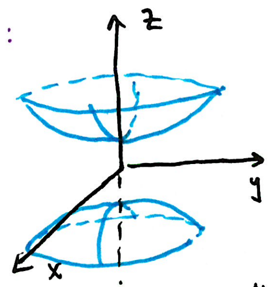
- Quadric Surfaces 3D applet: www.geogebra.org/m/VunKpsBA

- Extra Examples:

#37-40 use the 3D applet to sketch the following by identifying the quadric surface and the values of a, b, c. [Note may not be centered on applet]

#37 $-4x^2-y^2+z^2=1$

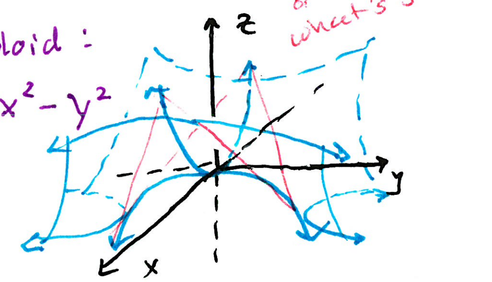
Hyperboloid of 2 sheets: $-\frac{x^2}{(1/2)^2}-\frac{y^2}{1^2}+\frac{z^2}{1^2}=1$



#38 $x^2-y^2-z=0$

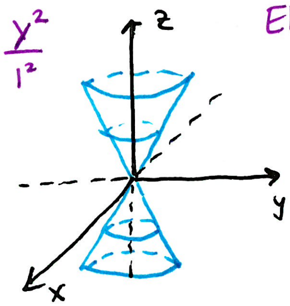
Hyperbolic Paraboloid: $z=x^2-y^2$

or in red is what's shown



#39 $-4x^2-y^2+z^2=0$

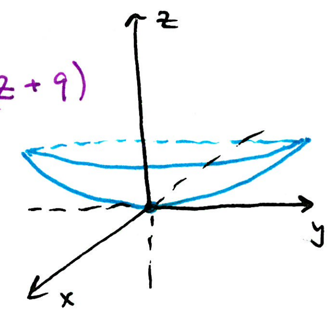
Cone: $z^2=\frac{x^2}{(1/2)^2}+\frac{y^2}{1^2}$



#40 $x^2-6x+4y^2-z=0$

Elliptic Paraboloid: $(x-3)^2+4y^2=(z+9)$

Center $(3,0,-9)$



#47 Traditionally, the earth's surface has been modeled as a sphere, but the World Geodetic System uses an ellipsoid as a more accurate model. It places the earth's center at the origin and north pole on the z-axis. The distance from the center to the poles is 6356.523 km and the distance to the equator is 6378.137 km.

(a) Find the model $\frac{x^2}{(6378.137)^2}+\frac{y^2}{(6378.137)^2}+\frac{z^2}{(6356.523)^2}=1$

(b) Curves of equal latitude are traces in the planes $z=k$. What are the curves?

$\boxed{\text{Circles}}$ since the coefficients in front of x^2, y^2 are equal

(c) Meridians (curves of equal longitude) are traces in planes $y=mx$. What are they?

$$\frac{(m^2+1)x^2}{(6378.137)^2}+\frac{z^2}{(6356.523)^2}=1 \Rightarrow \boxed{\text{Ellipses}}$$