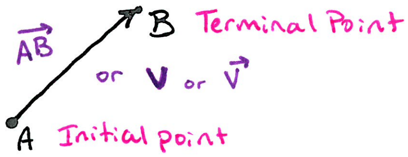
# Section 12.2 - Vectors





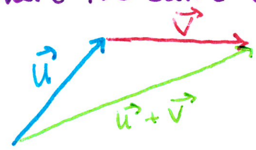


$|\vec{v}|$: length of vector

Vectors have a length and direction

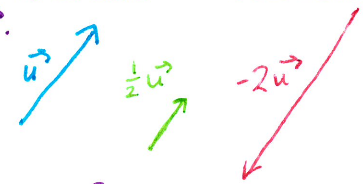
**Equivalent**: only if the vectors have the same direction and length

**Addition**: Sum Components




Place second where first ends to create new vector

**Scalar Multiplication**: $c$ a Scalar ($c \in \mathbb{R}$), $\vec{v}$ a vector then $c\vec{v}$ is a vector in the direction of $\vec{v}$ with length $c$ times length of $\vec{v}$.




- Vector Components:

Vectors in $\mathbb{R}^2$: $V^2$

$\vec{v} = \langle x, y \rangle$

Vectors in $\mathbb{R}^3$: $V^3$

$\vec{v} = \langle x, y, z \rangle$

Vectors in $\mathbb{R}^n$: $V^n$

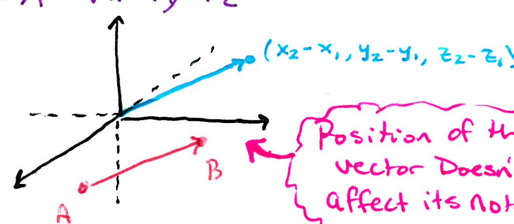
$\vec{v} = \langle x_1, x_2, \dots, x_n \rangle$

- Magnitude (length) of a vector:

$V^2$: $|\vec{v}| = |\langle x, y \rangle| = \sqrt{x^2 + y^2}$ $\qquad$ $V^3$: $|\vec{v}| = |\langle x, y, z \rangle| = \sqrt{x^2 + y^2 + z^2}$

- **Position vector**: $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

$\vec{v} = \vec{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$




Position of the vector Doesn't affect its notation

- Operations of vectors:

$\langle a, b \rangle + \langle c, d \rangle = \langle a+c, b+d \rangle$

$\langle a, b \rangle - \langle c, d \rangle = \langle a-c, b-d \rangle$

$c\langle a, b \rangle = \langle ca, c \cdot b \rangle$

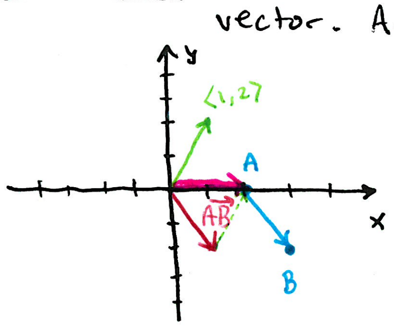
All computed componentwise

- Properties of vectors:

1. $\vec{v} + \vec{u} = \vec{u} + \vec{v}$ Commutative
2. $\vec{v} + (\vec{u} + \vec{w}) = (\vec{v} + \vec{u}) + \vec{w}$ Associative
3. $\vec{v} + \vec{0} = \vec{v}$ additive identity
4. $\vec{v} + (-\vec{v}) = \vec{0}$ additive inverse
5. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
6. $(c+d)\vec{u} = c\vec{u} + d\vec{u}$
7. $(cd)\vec{u} = c(d\vec{u}) = d(c\vec{u})$

   (5–7) Distributive Laws
8. $1 \cdot \vec{v} = \vec{v}$ Scalar Identity

**Example** Sketch the vector between $A(2,0)$ and $B(3,-2)$. Sketch the position vector. Add the vector $\langle 1, 2 \rangle$ to the vector $\vec{AB}$. Sketch it.




$\vec{AB} = \langle 1, -2 \rangle$

$\vec{AB} + \langle 1, 2 \rangle = \boxed{\langle 2, 0 \rangle}$

# Section 12.2 - Vectors

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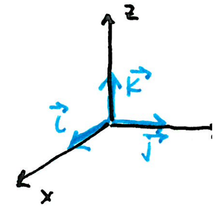
**Example 4** $\vec{a} = \langle 4, 0, 3 \rangle$ $\vec{b} = \langle -2, 1, 5 \rangle$ Find $|\vec{a}|$ and $2\vec{a} + 5\vec{b}$

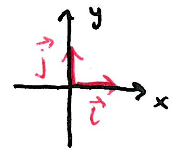
$$|\vec{a}| = \sqrt{4^2 + 0^2 + 3^2} = \boxed{5}$$

$$2\vec{a} + 5\vec{b} = 2\langle 4, 0, 3 \rangle + 5\langle -2, 1, 5 \rangle = \langle 8 - 10,\ 0 + 5,\ 6 + 25 \rangle = \boxed{\langle -2, 5, 31 \rangle}$$

- **Standard Basis Vectors**: $\vec{i} = \langle 1, 0, 0 \rangle$ $\vec{j} = \langle 0, 1, 0 \rangle$ $\vec{k} = \langle 0, 0, 1 \rangle$ in $V^3$

  $\vec{i} = \langle 1, 0 \rangle$ $\vec{j} = \langle 0, 1 \rangle$ in $V^2$




- **Unit Vector**: vector whose length is 1; vector $\vec{u}$ in direction of $\vec{v}$, unit

$$\boxed{\vec{u} = \frac{\vec{v}}{|\vec{v}|}}$$

Dividing/multiplying by a scalar changes the length by that scalar value (+).

**Example** Express $\vec{v} = \langle 2, -1, -2 \rangle$ in terms of $\vec{i}, \vec{j}, \vec{k}$ and find a unit vector for $\vec{v}$.

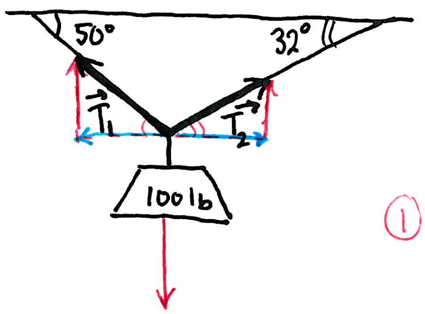
$$\vec{v} = \langle 2, 0, 0 \rangle + \langle 0, -1, 0 \rangle + \langle 0, 0, -2 \rangle = \boxed{2\vec{i} - \vec{j} - 2\vec{k}}$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 2, -1, -2 \rangle}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{1}{3}\langle 2, -1, -2 \rangle = \boxed{\left\langle \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3} \right\rangle}$$

- **Applications**: velocity and acceleration in space (3D), forces

  Recall: Resultant Force is the sum of forces acting on an object.

**Example 7** 100 lb weight hangs from two wires. Find the tension forces $\vec{T_1}$ and $\vec{T_2}$.




Resultant of Vertical/horizontal Forces are 0

Vertical Forces:

① $|\vec{T_1}| \sin 50° + |\vec{T_2}| \sin 32° = 100$

Horizontal Forces:

② $|\vec{T_1}| \cos 50° = |\vec{T_2}| \cos(32°)$

From ②: $|\vec{T_2}| = \dfrac{|\vec{T_1}| \cos 50°}{\cos 32°}$ Sub in ①: $|\vec{T_1}| \sin 50° + |\vec{T_1}| \cos 50° \tan 32° = 100$

Solve for $|\vec{T_1}|$: $|\vec{T_1}| = \dfrac{100}{\sin 50° + \cos 50° \tan 32°} \approx 85.64 \text{ lb}$

$$|\vec{T_2}| \approx \frac{85.64 \cos 50°}{\cos 32°} \approx 64.91 \text{ lb}$$

$$\vec{T_1} = \langle -85.64 \cos 50°,\ 85.64 \sin 50° \rangle \approx \boxed{\langle -55.05, 65.60 \rangle}$$

$$\vec{T_2} = \langle 64.91 \cos 32°,\ 64.91 \sin 32° \rangle \approx \boxed{\langle 55.05, 34.40 \rangle}$$

# Section 12.2 - Vectors



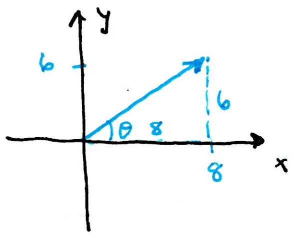
- Extra Examples:

**#26** Find a vector that has the same direction as $\langle -2, 4, 2\rangle$ but length 6.

$|\vec{v}| = \sqrt{2^2+4^2+2^2} = \sqrt{24} = 2\sqrt{6}$ Multiply $\vec{v}$ by $\frac{6}{2\sqrt{6}} = \frac{\sqrt{6}}{2}$

$\boxed{u = \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6}\rangle}$ Check: $|\vec{u}| = \sqrt{\sqrt{6}^2 + (2\sqrt{6})^2 + \sqrt{6}^2} = \sqrt{6+4\cdot 6+6} = 6$ ✓

**#28** What is the angle between $8\vec{i}+6\vec{j}$ and the positive x-axis?



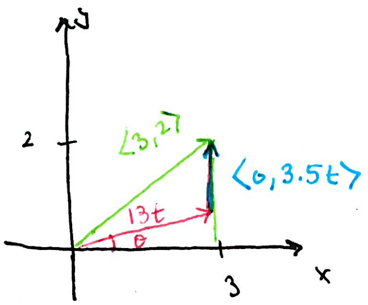

$\tan\theta = \frac{6}{8}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) \approx \boxed{36.87°}$

**#39** A boat man wants to cross a river that is 3 km wide and land 2 km upstream. The current is 3.5 km/hr and the boat speed is 13 km/hr.

(a) In what direction should he steer?

(b) How long will the trip take?




Boat vector: $\langle 13t\cos\theta, 13t\sin\theta\rangle$

+ Water vector: $\langle 0, 3.5t\rangle$

= Crossing: $\langle 3, 2\rangle$

① $3 = 13t\cos\theta$

② $2 = 3.5t + 13t\sin\theta$

From ①: $t = \frac{3}{13\cos\theta}$

Sub in ②: $2 = \frac{10.5}{13\cos\theta} + 3\tan\theta$

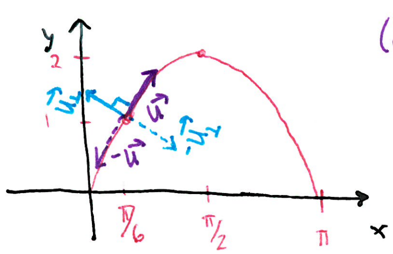
(a) $\boxed{\theta \approx 20.74°}$

(b) $t \approx \frac{3}{13\cos 20.74°} \approx \boxed{0.247\text{ hr}}$

**#42** (a) Find the unit vectors that are parallel to the tangent line to $y = 2\sin x$ at $(\pi/6, 1)$.

(b) Find the unit vectors $\perp$ to the tangent line.

(c) Sketch $y = \sin x$ and the vectors from (a), (b) at $(\pi/6, 1)$.




(a) $y' = 2\cos(x)$ $y'(\pi/6) = \sqrt{3}$

Slope over x up y (rise over run)

Vector in direction of tangent $\vec{v} = \langle 1, \sqrt{3}\rangle$

$|\vec{v}| = \sqrt{1+3} = 2$ $\vec{u} = \frac{1}{2}\langle 1, \sqrt{3}\rangle = \boxed{\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}\right\rangle}$ $-\vec{u} = \boxed{\left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\rangle}$

(b) Tangent slope: $\sqrt{3}$ $\perp$ slope: $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\vec{v}_\perp = \langle -3, \sqrt{3}\rangle$ $|\vec{v}_\perp| = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$\vec{u}_\perp = \boxed{\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle}$ and $-\vec{u}_\perp = \boxed{\left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2}\right\rangle}$