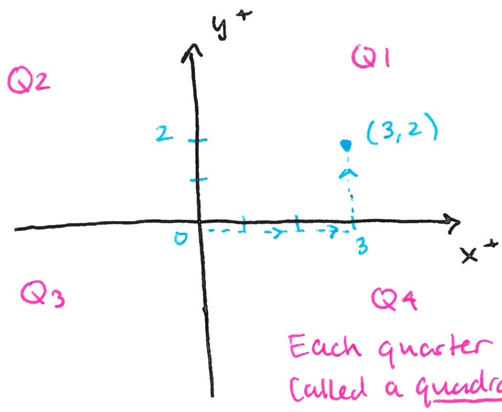
Section 12.1 – 3D Coordinate System

MVC

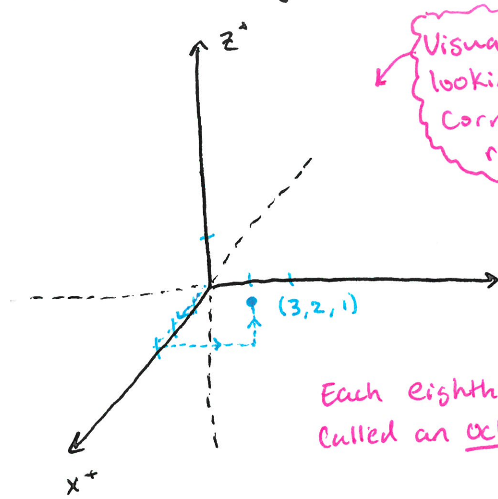
- 2D - Cartesian Coordinate System



Each quarter called a <u>quadrant</u>

- 3D - Coordinate system

Visualize by looking at the corner of a room



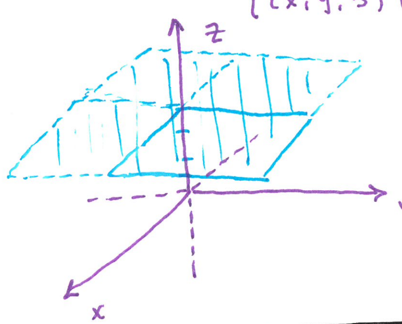
Each eighth called an <u>octant</u>

2D	3D
Point: (x, y)	Point: (x, y, z)
Sketch: $(3,2)$	Sketch: $(3,2,1)$
Set: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x, y \in \mathbb{R}\}$	Set: $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$
Equations: of x, y called <u>curves</u>	Equations: x, y, z called <u>surfaces</u>

Example 1 What surfaces in $\mathbb{R}^3$ are represented by the equations:

(a) $z = 3$

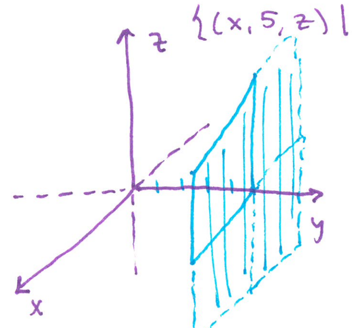
$\{(x, y, 3) \mid x, y \in \mathbb{R}\}$



Flat plane up 3 units parallel to xy-plane

(b) $y = 5$

$\{(x, 5, z) \mid x, z \in \mathbb{R}\}$



Vertical plane parallel to xz-plane over 5 units

★ Visit: www.math.uri.edu/~bkaskosz/flashmo/graph3d2/

- Distance Between two Points P_1 & P_2:

$\mathbb{R}^2$: $P_1(x_1, y_1)$ $P(x_2, y_2)$

$$D = |P_1P_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

<u>From Pythagorean's Identity</u>

$\mathbb{R}^3$: $P_1(x_1, y_1, z_1)$ $P_2(x_2, y_2, z_2)$

$$D = |P_1P_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

Recall length of the diagonal in a solid rectangular box – Pythagorean's Identity applied twice

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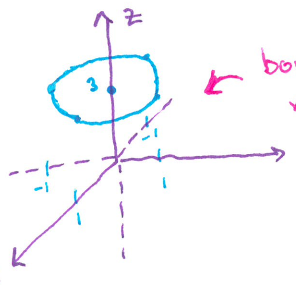
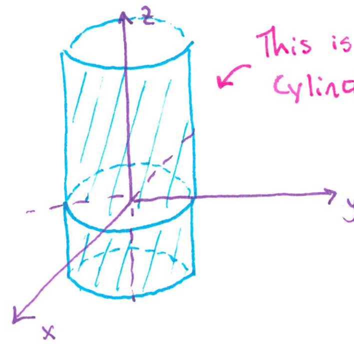
Example 2 (a) Which points (x,y,z) satisfy $x^2+y^2=1$ and $z=3$? Sketch
(b) What does the equation $x^2+y^2=1$ represent in $\mathbb{R}^3$? Sketch

(a) Points only on the plane $z=3$
In a circle of radius 1 about $(0,0,3)$



boundary of circle not solid

(b) a circle of radius 1 for every plane $z=k \Rightarrow$ Cylinder along z-axis



This is a hallow cylinder not solid

- Equation of a sphere:

Recall: A circle is the set of all points in $\mathbb{R}^2$ equidistant from the center.
A sphere is the set of all points in $\mathbb{R}^3$ equidistant from the center.

Circles:		Spheres:
$x^2+y^2=r^2$	Radius r, center O	$x^2+y^2+z^2=r^2$
$(x-h)^2+(y-k)^2=r^2$	Radius r; Center P $P(h,k)$ $P(h,k,\ell)$	$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$

Example Show $x^2+y^2+z^2=-4x$ is the equation of a sphere. Sketch

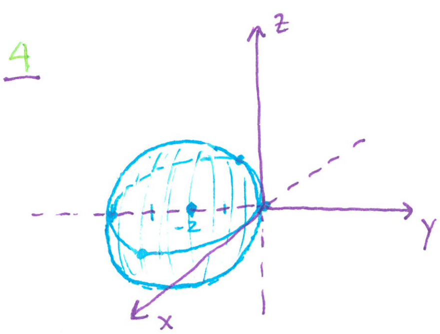
Complete the square: $\left(\frac{(\text{Coefficient of } x)}{2}\right)^2$

$$(x^2+4x+4)+y^2+z^2=0+4$$

$$(x+2)^2+y^2+z^2=2^2$$

radius: 2

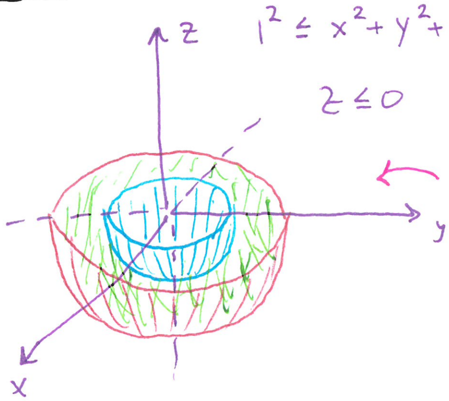
Center: $(-2,0,0)$



Example 7 What region in $\mathbb{R}^3$ is represented by $1\le x^2+y^2+z^2\le 4$ and $z\le 0$? Sketch

$1^2\le x^2+y^2+z^2\le 2^2$ $\begin{cases}\text{all points outside sphere of radius 1} \\ \text{but inside sphere of radius 2}\end{cases}$

$z\le 0$ $\to$ Bottom halves



Think advacado halved, pitted ☺

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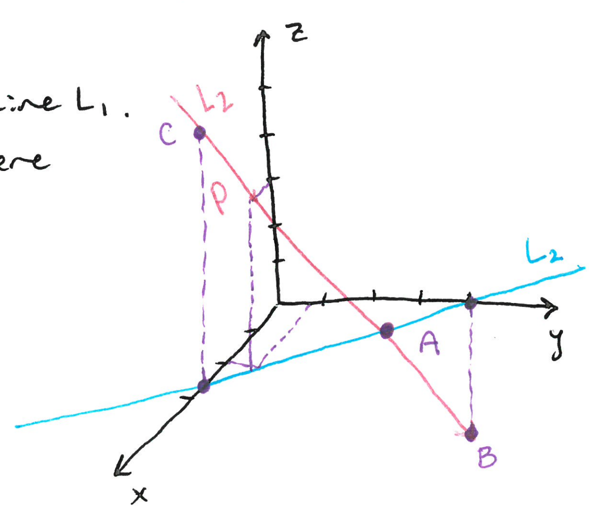
- Extra Examples:

\# 39. The figure shows a line L_1 in space, a second line L_2 is the projection of L_1 onto the xy-plane.

(a) Find the coordinates of the point P on the line L_1.

(b) Locate on the diagram the points A, B, C where L_1 intesects the xy, yz, zx planes.



(a) $(2,1,3) = P$

(b) See graph →

\# 41. Find an equation of the set of all points equidistant from the points $A(-1,5,3)$ and $B(6,2,-2)$. Describe the set.

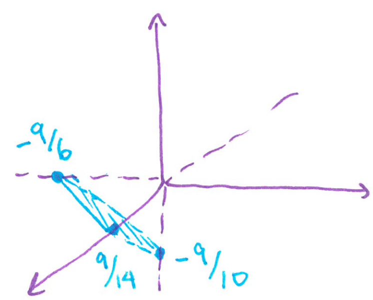
$$\sqrt{(x+1)^2+(y-5)^2+(z-3)^2} = \sqrt{(x-6)^2+(y-2)^2+(z+2)^2}$$

$$(x+1)^2+(y-5)^2+(z-3)^2 = (x-6)^2+(y-2)^2+(z+2)^2$$

$$\cancel{x^2}+2x+1+\cancel{y^2}-10y+25+\cancel{z^2}-6z+9 = \cancel{x^2}-12x+36+\cancel{y^2}-4y+4+\cancel{z^2}+4z+4$$

$$14x-6y-10z=9$$

This is the equation of a plane



\# 43. Find the distance between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=4x+4y+4z-$

$$(x-2)^2+(y-2)^2+(z-2)^2 = -11+4\times 3 = 1$$

Center: $(2,2,2)$ radius: 1

$$x^2+y^2+z^2=4$$

Center: $(0,0,0)$ radius: 2

distance between radi

subtract radi lengths

$$\text{Distance} = \sqrt{(2-0)^2+(2-0)^2+(2-0)^2} - 2 - 1$$

$$= \boxed{\sqrt{12}-3}$$