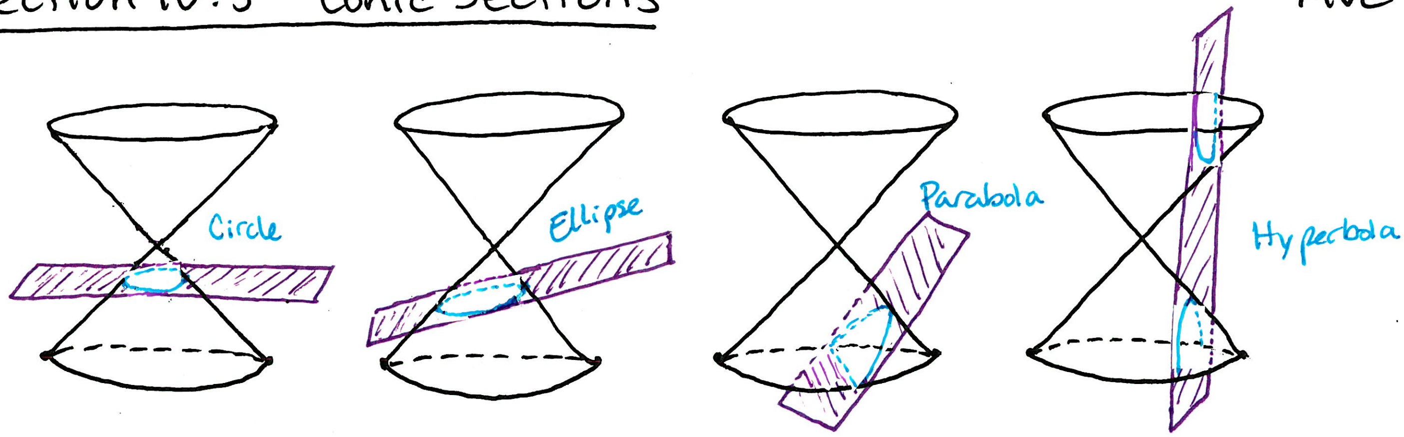
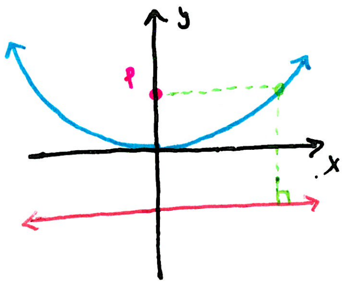
Section 10.5 - Conic Sections

MVC



- **Parabolas**: Set of all points in a plane equidistant from a point (focus) and a line (directrix)

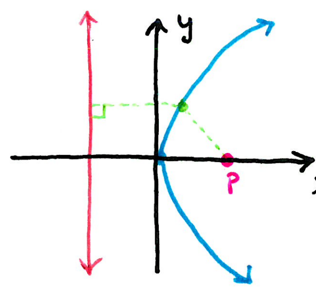


Equation: $y = \frac{1}{4p}x^2$

focus: $(0, p)$

Directrix: $y = -p$

vertex: $(0,0)$



Equation: $x = \frac{1}{4p}y^2$

focus: $(p, 0)$

Directrix: $x = -p$

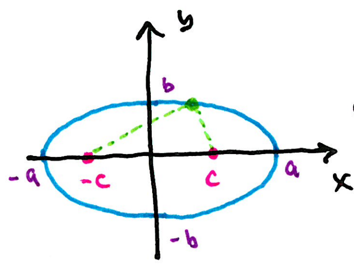
vertex: $(0,0)$

Standard form: $y - k = \frac{1}{4p}(x-h)^2$ $\qquad$ $x - h = \frac{1}{4p}(y-k)^2$

Vertex at (h,k)

General form: $ax^2 + bx + cy + d = 0$ $\qquad$ $ay^2 + by + cx + d = 0$

- **Ellipses**: Set of all points in a plane such that the sum of a point's distances between itself and two fixed points (foci) remains constant.

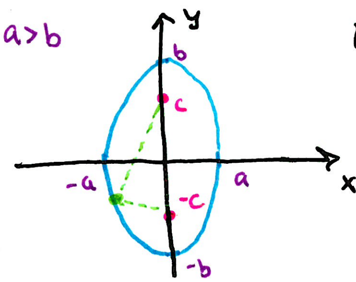


Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad a > b$

Center: $(0,0)$

foci: $(\pm c, 0)$

$a^2 = b^2 + c^2$



Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad b > a$

Center: $(0,0)$

foci: $(0, \pm c)$

$b^2 = a^2 + c^2$

Standard form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Center (h,k)

General form: $ax^2 + by^2 + cx + dy + e = 0$

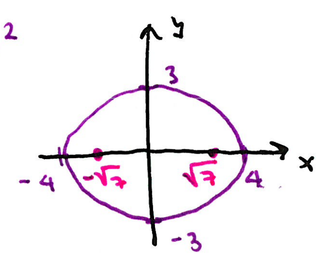
Example Identify, sketch, find foci (focus): a) $144 - 9x^2 - 16y^2 = 0$ $\quad$ b) $9x + 4y^2 = 36$

(a) $144 = 9x^2 + 16y^2$

$1 = \frac{x^2}{4^2} + \frac{y^2}{3^2}$

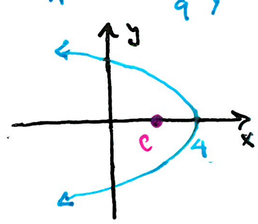
Ellipse

$\boxed{c = \pm\sqrt{7}}$



(b) $x = -\frac{4}{9}y^2 + 4$ $\qquad$ $4p = -\frac{9}{4}$ $\quad$ $p = -\frac{9}{16}$

$\boxed{c = 4 + -\frac{9}{16}}$



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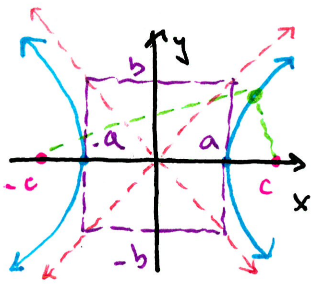
- **Hyperbolas**: set of all points in a plane such that the absolute value of the difference between a points distances between itself and two fixed points (foci) remains constant.



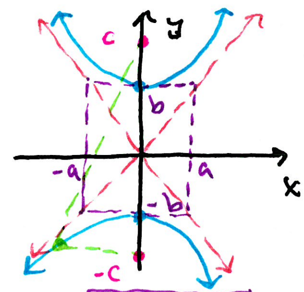
Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

foci: $(\pm c, 0)$

Asymptotes: $y = \pm \frac{b}{a}x$

Center: $(0,0)$

vertices: $(\pm a, 0)$



$\boxed{c^2 = a^2 + b^2}$

Equation: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

foci: $(0, \pm c)$

Asymptotes: $y = \pm \frac{b}{a}x$

Center: $(0,0)$

vertices: $(0, \pm b)$

Standard form:
Center (h,k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

General form: $ax^2 - by^2 + cx + dy + e = 0 \qquad -ax^2 + by^2 + cx + dy + e = 0$

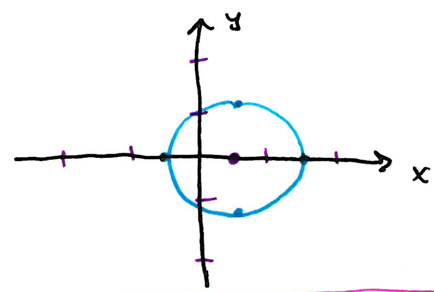
Example Identify, sketch, find foci, vertices, center and any asymptotes.

① $6x^2 - 6x + 6y^2 = 9/2$

Circle: $x^2 - x + y^2 = \frac{3}{4}$

$(x^2 - 1x + \frac{1}{4}) + y^2 = 1$

$(x - \frac{1}{2})^2 + y^2 = 1$



$\boxed{\text{Center: } (\frac{1}{2}, 0) \quad \text{radius} = 1}$

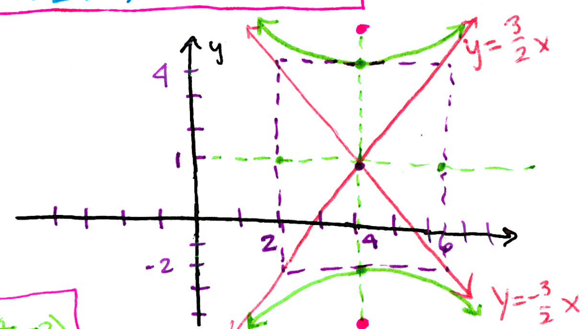
② $9x^2 - 4y^2 - 72x + 8y = -176$

Hyperbola: $9(x^2 - 8x + 16) - 4(y^2 - 2y + 1) = -176 + 9(16) - 4(1)$

$9(x-4)^2 - 4(y-1)^2 = -36$

$\frac{(y-1)^2}{3^2} - \frac{(x-4)^2}{2^2} = 1$

$\boxed{\text{Center } (4,1) \quad \text{vertices } (4,4)(4,-2)}$



$c^2 = 3^2 + 2^2 \qquad c = \sqrt{13}$

$\boxed{\text{foci: } (4, 1 \pm \sqrt{13})}$

Example Consider $ax^2 + by^2 + cx + dy + e = 0$
What must be true about a, b, c, d, e to have:

① Circle

$a = b$

$-\frac{e}{a} + \frac{c^2}{4a^2} + \frac{d^2}{4a^2} > 0$

② Ellipse

Same sign on $a, b \neq 0$

$-e + \frac{c^2}{4a} + \frac{d^2}{4b} > 0$

③ Parabola

a or $b = 0$ not both

$c \neq 0 \quad d \neq 0$

No other restrictions

④ Hyperbola

a, b opposite sign $\neq 0$

$-e + \frac{c^2}{4a} + \frac{d^2}{4b} \neq 0$

⑤ line

$a = b = 0 \quad c \neq 0$ and $d \neq 0$

No other restrictions

⑥ No Solutions

See ①-⑤

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- Extra Examples:

★ Find parametric equations for the standard form of each Conic section.

① Circle: $(x-h)^2+(y-k)^2=r^2$ ← Check $\boxed{\begin{array}{l} x=h+r\cos\theta \\ y=k+r\sin\theta \end{array}}$

② ellipse: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ ← Check $\boxed{\begin{array}{l} x=h+a\cos\theta \\ y=k+b\sin\theta \end{array}}$

$\Big\}\ \cos^2\theta+\sin^2\theta=1$

③ Parabola: $y-k=\frac{1}{4p}(x-h)^2$ y function of x → $\boxed{\begin{array}{l} x=t \\ y=\frac{1}{4p}(t-h)^2+k \end{array}}$

④ Hyperbola: $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ $\boxed{\begin{array}{l} x=h+a\sec\theta \\ y=k+b\tan\theta \end{array}}$ $\rightarrow \sec^2\theta-\tan^2\theta=1$

\# 55 Determine the type of curve represented by the equation:

$$\frac{x^2}{k}+\frac{y^2}{k-16}=1$$

in each case (a) $k>16$ (b) $0<k<16$ (c) $k<0$

(a) $k>16$ then $k-16>0$ $\Rightarrow$ $\boxed{\text{Ellipse}}$ centered at $(0,0)$ with vertices $(\pm\sqrt{k},0)$ and $(0,\pm\sqrt{k-16})$

(b) $0<k<16$ $-16<k-16<0$ $\Rightarrow$ $\boxed{\text{Hyperbola}}$ about x-axis vertices $(\pm\sqrt{k},0)$ asymptotes $y=\pm\frac{\sqrt{(k-16)}}{\sqrt{k}}x$

(c) $k<0$ $\Rightarrow$ $\boxed{\text{Hyperbola}}$ about x-axis vertices $(\pm\sqrt{k},0)$

\# 56 (a) Show that the equation of the tangent line to the parabola $y^2=4px$ at the point (x_0,y_0) can be written as $y_0y=2p(x+x_0)$

(b) What is the x-intercept of this tangent line?

(a) $2y\frac{dy}{dx}=4p \Rightarrow \frac{dy}{dx}=\frac{4p}{2y} \Rightarrow \left.\frac{dy}{dx}\right|_{(x_0,y_0)}=\frac{2p}{y_0}$

$y-y_0=\frac{2p}{y_0}(x-x_0) \Rightarrow y_0y=2px-2px_0+y_0^2$

$=2px-2px_0+4px_0$

$=2p(x+x_0)$ ✓

(b) $y=\frac{2p}{y_0}x-\frac{2px_0}{y_0}+y_0=\frac{2px}{y_0}+\frac{y_0^2-2px_0}{y_0}=\frac{2px}{y_0}+\frac{2px_0}{y_0}$ $\boxed{\left(0,\frac{2px_0}{y_0}\right)}$