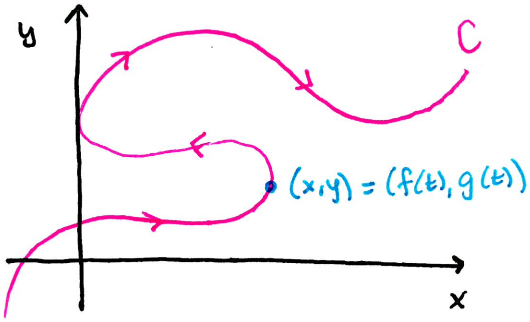
Section 10.1 - Parametric Equations

MVC

Not all curves in the xy-plane can be written as a function of x or as a function of y - Example is a circle.



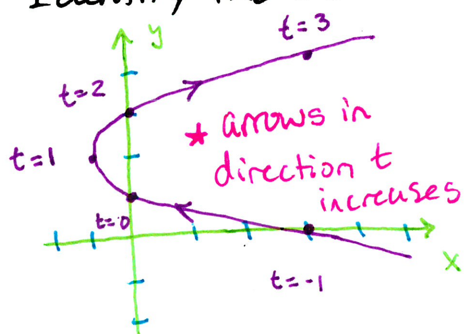
A bigger collection of curves, other than functions, where points are functions of a variable called a parameter.

Parametric Equations: $x=f(t)$, $y=g(t)$, $z=h(t)$, ... t the parameter

Parametric Curve: $C=\{(x,y,\ldots)=(f(t),g(t),\ldots) \mid t\in D\subseteq\mathbb{R}\}$

Example 1 Sketch and Identify the curve defined by $x=t^2-2t$, $y=t+1$.

t	x	y
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4



* arrows in direction t increases

Identify: $t=y-1$

$$\boxed{x=(y-1)^2-2(y-1)}$$

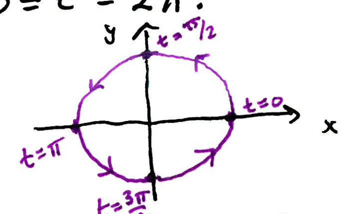
$$x=(y-2)^2-1$$

$\boxed{\text{Parabola, vertex } (-1,2)}$

Example 2 What curve is represented by the following parametric equations $x=\cos t$ $y=\sin t$ $0\le t\le 2\pi$?

$$x^2+y^2=\cos^2 t+\sin^2 t=1$$

↑ unit circle centered at $(0,0)$



Example 4 Find parametric equations for the circle of radius r and center (h,k).

Radius r: $r^2=r^2\cos^2 t+r^2\sin^2 t$

mult ↑ by r^2

$x=r\cos t$ $y=r\sin t$

Center (h,k): $\boxed{x=h+r\cos t \quad y=k+r\sin t}$

move x by +h, y by +k

Example 6 Use your graphing calculator to graph $x=y^4-3y^2$

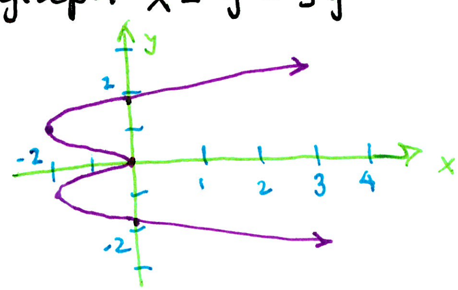
Parametric Equations: $y=t$

$x=t^4-3t^2$

$\boxed{\text{Mode}}$ → $\boxed{\text{PAR}}$ → $\boxed{\text{Y=}}$

$X_1=x(t)$

$Y_1=y(t)$



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- Sketch on your graphing Calculator

① $x = \sin t - \sin 2.3t$
$y = \cos t$
$0 \le t \le 70$

$-2 \le x \le 2$
$-1 \le y \le 1$
Small Tstep size

② $x = 16\sin^3 t$
$y = 13\cos t - 5\cos(2t) - 2\cos(3t) - \cos(4t)$

$0 \le t \le 2\pi$
$-20 \le x, y \le 20$

[Use TI-84 Emulator or Wolfram Alpha for graphs]

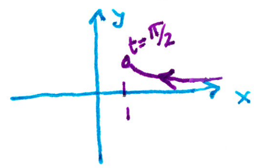
Extra Examples

#13 (a) Eliminate the parameter (b) sketch the curve

$x = \sin t \quad y = \csc t \quad 0 < t < \pi/2$

(a) $y = \dfrac{1}{\sin t} = \boxed{\dfrac{1}{x}}$

(b)

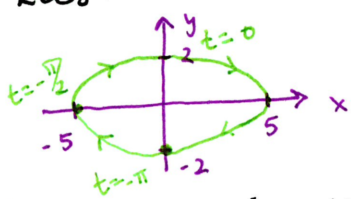


#21 Describe the motion of a particle with position (x, y):

$x = 5\sin t \quad y = 2\cos t \quad -\pi \le t \le 5\pi$

$\left(\dfrac{x}{5}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$

This is an Ellipse



Begin at $(0,-2)$ move clockwise around Ellipse $x^2/5^2 + y^2/2^2 = 1$ 3 times.

#31(a) Show $x = x_1 + (x_2 - x_1)t \quad y = y_1 + (y_2 - y_1)t$ where $0 \le t \le 1$ describes the line segment between (x_1, y_1) and (x_2, y_2).

Eliminate t: $\dfrac{x - x_1}{x_2 - x_1} = t$

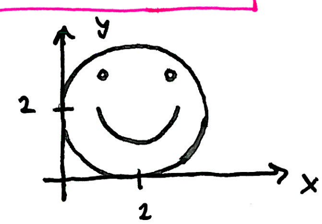
Sub t into y: $y = y_1 + (y_2 - y_1)\left(\dfrac{x - x_1}{x_2 - x_1}\right)$

Rearrange:

$$\boxed{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)}$$

Point slope form of line

#35 Use a graphing calculator to reproduce the graph



Face: $x = 2 + 2\cos t$ — radius 2
$y = 2 + 2\sin t$
center $(2,2)$

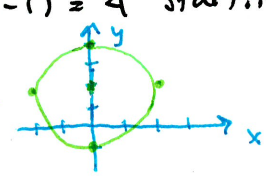
Eyes: $x = 1 + 0.1\cos t$ — radius 0.1
$y = 3 + 0.1\sin t$
$x = 3 + 0.1\cos t$
$y = 3 + 0.1\sin t$

Mouth: $x = 2 - \cos \frac{1}{2}t$
$y = 2 - \sin \frac{1}{2}t$

makes period 4π so half is sketched

#33(c) Find parametric equations that travel halfway counterclockwise around $x^2 + (y-1)^2 = 4$ starting at $(0, 3)$.

Center: $(0, 1)$
radius: 2



Clockwise at $(0,3)$: $x = \sin t$, $y = \cos t$

Counter: $x = -\sin t$, $y = \cos t$

halfway: $\boxed{x = -\sin \frac{1}{2}t \quad y = \cos \frac{1}{2}t}$