Section 16.9 Homework

1. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}$ on the region E given, $\mathbf{F}(x, y, z) = \langle 3x, xy, 2xz \rangle$, and E is the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

3. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}$ on the region E given, $\mathbf{F}(x, y, z) = \langle xy, yz, zx \rangle$, and E is the solid cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$.

6. Use the Divergence Theorem to calculate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, that is calculate the flux of $\mathbf{F}$ across S, given $\mathbf{F}(x, y, z) = \langle x^2z^3, 2xyz^3, xz^4 \rangle$, and S is the surface of the box with vertices $(\pm 1, \pm 2, \pm 3)$.

9. Use the Divergence Theorem to calculate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, that is calculate the flux of $\mathbf{F}$ across S, given $\mathbf{F}(x, y, z) = \langle xy \sin z, \cos(xz), y \cos z \rangle$, and S is the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.

12. Use the Divergence Theorem to calculate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, that is calculate the flux of $\mathbf{F}$ across S, given $\mathbf{F}(x, y, z) = \langle x^4, -x^3z^2, 4xy^2z \rangle$, and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 2$ and $z = 0$.

13. Use the Divergence Theorem to calculate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, that is calculate the flux of $\mathbf{F}$ across S, given $\mathbf{F}(x, y, z) = \langle 4x^3z, 4y^3z, 3z^4 \rangle$, and S is the sphere with radius R and center the origin.

23. Verify that $\div E = 0$ for the electric field $E(\mathbf{x}) = \frac{\epsilon Q}{|\mathbf{x}|^3}\mathbf{x}$.