Answers to Worksheet 55 - Optimization Problems

1) A = the area of the composite window x = the width of the bottom window = the diameter of the top window

Function to maximize: $A = x\left(\frac{14}{2} - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$ where $0 < x < \frac{56}{4 + \pi}$

Dimensions of the bottom window: $\frac{28}{4 + \pi}$ ft (width) by $\frac{14}{4 + \pi}$ ft (height)

2) L = the total length of rope x = the horizontal distance from the short pole to the stake

Function to minimize: $L = \sqrt{x^2 + 12^2} + \sqrt{(21 - x)^2 + 16^2}$ where $0 \le x \le 21$

Stake should be placed: 9 ft from the short pole (or 12 ft from the long pole)

3) p = the profit per day x = the number of items manufactured per day

Function to maximize: $p = x(160 - 0.05x) - (80x + 7000)$ where $0 \le x < \infty$

Optimal number of smartphones to manufacture per day: 800

4) V = the volume of the box x = the length of the sides of the squares

Function to maximize: $V = (16 - 2x)(10 - 2x)x$ where $0 < x < 5$

Sides of the squares: 2 in

5) d = the distance from point $(4, 0)$ to a point on the curve x = the x-coordinate of a point on the curve

Function to minimize: $d = \sqrt{(x - 4)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$

Point on the curve that is closest to the point $(4, 0)$: $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$

6) A = the area of the rectangle x = half the base of the rectangle

Function to maximize: $A = 2x\sqrt{8^2 - x^2}$ where $0 < x < 8$

Area of largest rectangle: 64