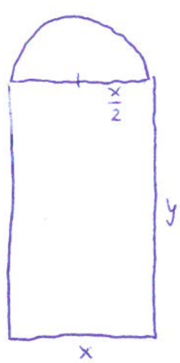
Pre-Calculus AB Name________________________________

Optimization Word Problems

Date________________ Period____

Solve each optimization problem.

1) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 14 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?



$2y + x + \pi \frac{x}{2} = 14$ $\quad y = 7 - \frac{x}{2} - \frac{\pi x}{4}$

$A = x \cdot y + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$ $\quad A = 7x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$

$0 < x < \frac{56}{4+\pi}$

$A = -\left(\frac{4+\pi}{8}\right)\left[x^2 - \left(\frac{56}{4+\pi}\right)x + \left(\frac{28}{4+\pi}\right)^2\right] + \frac{98}{4+\pi}$

$A = -\left(\frac{4+\pi}{8}\right)\left(x - \frac{28}{4-\pi}\right)^2 + \frac{98}{4+\pi}$

Largest Area: $\frac{98}{4+\pi}$ ft^2

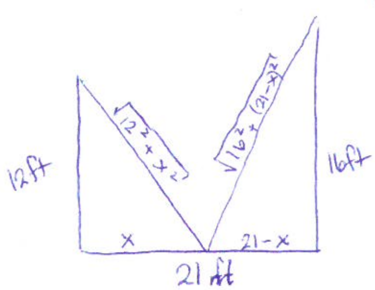
Dimensions:

Bottom: $\frac{28}{4+\pi}$ ft

height: $\frac{14}{4+\pi}$ ft

2) Two vertical poles, one 12 ft high and the other 16 ft high, stand 21 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

Amount of rope $R = \sqrt{12^2 + x^2} + \sqrt{16^2 + (21-x)^2}$ $\quad 0 \leq x \leq 21$



Graph $[0, 21] \times [0, 50]$

Minimum: $\boxed{x = 9\text{ ft} \quad R = 35\text{ ft}}$

3) A company has started selling a new type of smartphone at the price of \$160 – 0.05*x* where *x* is the number of smartphones manufactured per day. The parts for each smartphone cost \$80 and the labor and overhead for running the plant cost \$7000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

Revenue for x phones:
$x(160 - 0.05x)$

Cost for x phones:
$80x + 7000$

Profit for x phones:
$P = x(160 - 0.05x) - 80x - 7000$

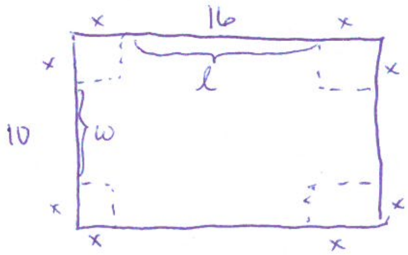
$P = -\frac{1}{20}x^2 + 80x - 7000 = -\frac{1}{20}(x^2 - 1600x + 640000) - 7000 + 32000$

$= -\frac{1}{20}(x - 800)^2 + 25{,}000$

Max Profit: \$25,000

\# of phones to sell: 800

4) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?



$V = (10-2x)(16-2x)\cdot x \quad 0 < x < 5$

Graph $[0,5] \times [0,200]$

Maximum Volume: 144 in^3

Dimensions:

Width: 6 in

length: 12 in

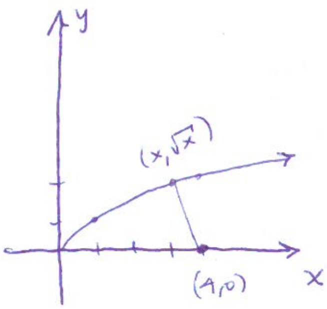
height: 2 in

$V = l \cdot w \cdot x$

$l = 10 - 2x$

$w = 16 - 2x$

5) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(4, 0)$?



$D = \sqrt{(4-x)^2 + (0-y)^2} \quad y = \sqrt{x}$

$D = \sqrt{(4-x^2) + (x)^2} \quad 0 \le x < \infty$

$D^2 = x^2 - 8x + 16 + x$

$D^2 = x^2 - 7x + 16$

$= \left(x^2 - 7x + \frac{49}{4}\right) + \frac{64}{4} - \frac{49}{4}$

$= \left(x - \frac{7}{2}\right)^2 + \frac{15}{4}$

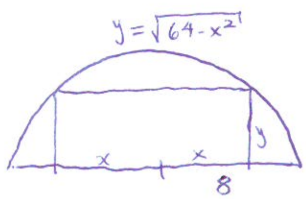
Min Distance is $\frac{\sqrt{15}}{2}$

$x = 7/2$

$y = \frac{\sqrt{14}}{2}$

6) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 8. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

$A = 2x \cdot y \quad y = \sqrt{64 - x^2}$



$A = 2x\sqrt{64 - x^2}$

$A^2 = 4x^2(64 - x^2)$

$A^2 = 256x^2 - 4x^4$

$A^2 = -4(x^4 - 64x^2 + 1024) + 4096$

$A^2 = -4(x^2 - 32)^2 + 4096$

Max Area : $\sqrt{4096} = 64 \text{ units}^2$

length : $2\sqrt{32} = 8\sqrt{2}$ units

width : $\sqrt{32} = 4\sqrt{2}$ units