Pre-Calc AB

Piecewise Functions

4/7/16

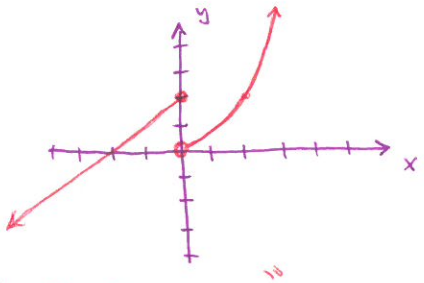
Topic: Piecewise Functions

- Evaluating
- Graphs
- Rules

★ Handout WS 1 on Piecewise Functions

Definition - A piecewise function consists of multiple functions defined over different intervals or subsets of the domain.

Example: $f(x) = \begin{cases} x+2 & \text{if } x \le 0 \\ \frac{1}{2}x^2 & \text{if } x > 0 \end{cases}$



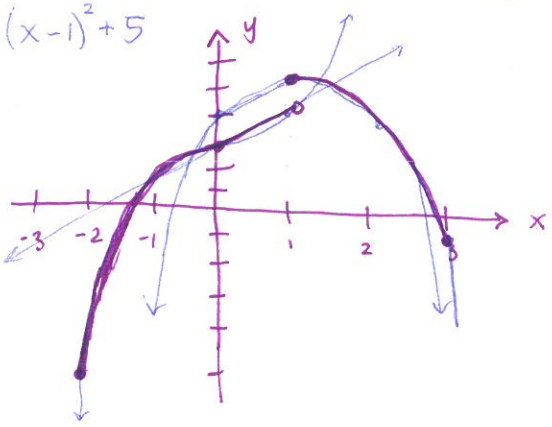
"$f(x) = x+2$ when $x \le 0$ and $f(x) = \frac{1}{2}x^2$ if $x > 0$."

Find $f(-3) = (-3)+2 = \boxed{-1}$ as $-3 \le 0$

$f(0) = (0)+2 = \boxed{2}$ as $0 \le 0$

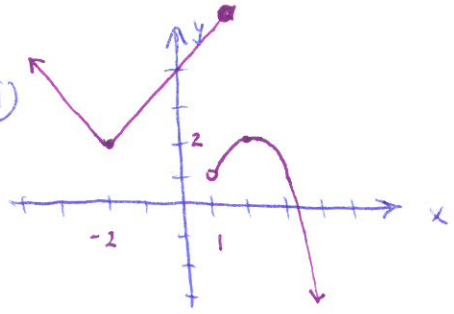
$f(3) = \frac{1}{2}(3)^2 = \boxed{\frac{9}{2}}$ as $3 > 0$

Example: Sketch the graph of $f(x) = \begin{cases} x^3+3 & \text{if } -2 \le x \le 0 \\ x+3 & \text{if } 0 < x < 1 \\ 4+2x-x^2 & \text{if } 1 \le x \le 3 \\ -(x-1)^2+5 \end{cases}$



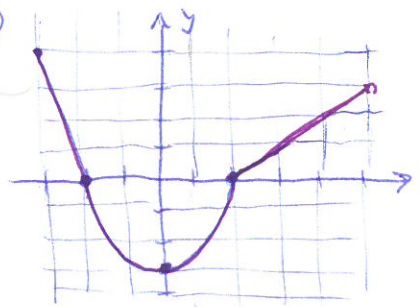
Example: Write a piecewise function for each graph:

1)



$f(x) = \begin{cases} |x+2|+2 & \text{if } x \le 1 \\ -(x-1)^2+2 & \text{if } x > 1 \end{cases}$

2)



$f(x) = \begin{cases} 4x-8 & \text{if } -3 \le x \le -2 \\ \frac{3}{4}(x-2)(x+2) & \text{if } -2 < x < 2 \\ x-2 & \text{if } 2 \le x < 5 \\ 4 & \text{if } x = 5 \end{cases}$

Pre-Calc AB

Piecewise Function

4/8/16

Topic: Piecewise Functions

- Step Functions
- Applications

★ Handout Piecewise Functions WS 2

The Greatest Integer Function: [Also called the Floor function for CS]

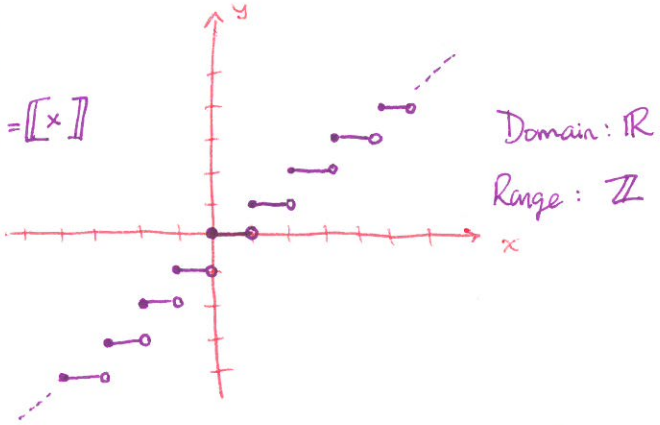
$$f(x) = [\![x]\!] = \begin{cases} x & \text{if } x \in \mathbb{Z} \\ \text{greatest Integer Less than } x & \text{if } x \notin \mathbb{Z} \end{cases}$$

$[\![4]\!] = 4$
$[\![-5]\!] = -5$
} in $\mathbb{Z}$

$[\![2.46]\!] = 2$] $2 < 2.46$
$[\![\pi]\!] = 3$] $3 < 3.1415\ldots$
$[\![-6\frac{1}{2}]\!] = -7$] $-7 < -6\frac{1}{2}$

Graph:

$f(x) = [\![x]\!]$



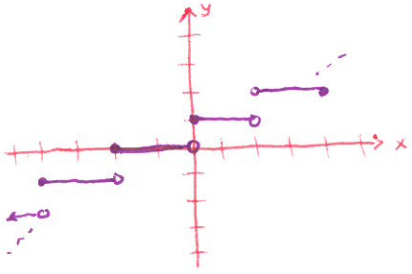
Domain: $\mathbb{R}$

Range: $\mathbb{Z}$

★ $f(x)$ is discontinuous at any integer value

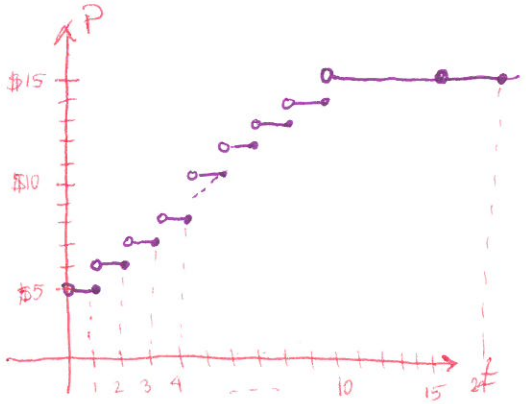
Example: Graph $f(x) = [\![\frac{1}{2}x + 1]\!] = [\![\frac{1}{2}(x+2)]\!]$

- Horizontal stretch by 2
- Horizontal shift left by 2



Example: Daantown Parking charges a $5 base fee for parking through 1 hour, and $1 for each additional hour or fraction thereof. The maximum fee for 24 hours is $15. Sketch a graph of the function and give a piecewise function that describes this pricing scheme.

$$P(t) = \begin{cases} [\![t]\!] + 5 & \text{if } 0 \le t \le 10 \\ 15 & \text{if } 10 < t \le 24 \end{cases}$$



Pre-Calc AB

Absolute Value

4/11/16

Topic: Absolute Value

- $|x|$
- $|f(x)|$
- $f(|x|)$

★ Handout Abs Value ws 1

Properties of Absolute Value: For all real Numbers a, b

1. $|ab| = |a| \cdot |b|$
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ provided $b \neq 0$
3. $|a| = |-a|$
4. Triangle Inequality $|a+b| \leq |a| + |b|$

Definitions:

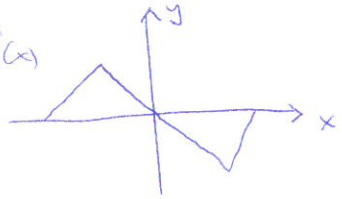
$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$f(|x|) = \begin{cases} f(x) & \text{if } x \geq 0 \\ f(-x) & \text{if } x < 0 \end{cases}$$

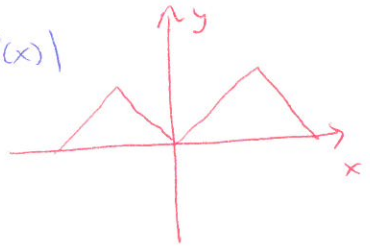
$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

Example:

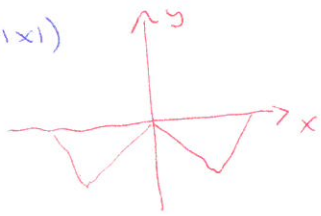
① $f(x)$



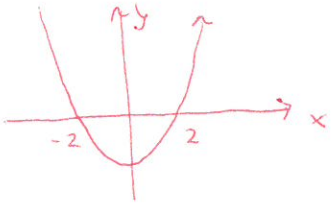
Graph $|f(x)|$



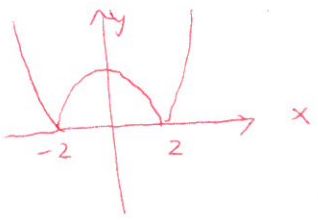
and $f(|x|)$



② $g(x) = x^2 - 4$



Graph $|f(x)|$



and $f(|x|)$