Pre-Calc AB

Functions

4/1/16 - 4/5/16

Topic: Functions

- Continuity, increasing, decreasing, max, mins,
- Domain of functions, evaluation
- even, odd functions
- Function operations

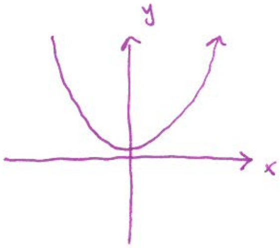
★ Handout Functions WS I

★ Quiz 13 on Wednesday
Exp. Functions
Function review

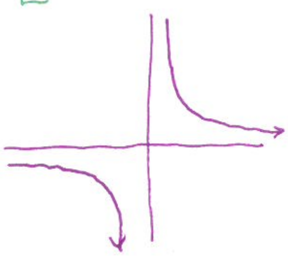
Definition - A function is a relation in which each element of the domain corresponds to exactly one element in the range. [Vertical line test]

We say a function is continuous over an interval of its domain if its left-hand graph can be sketched without lifting the pencil from the paper.

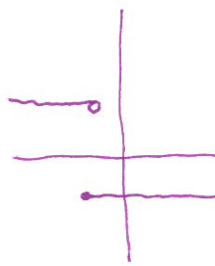
[No holes, No Asymptotes, No Jumps]



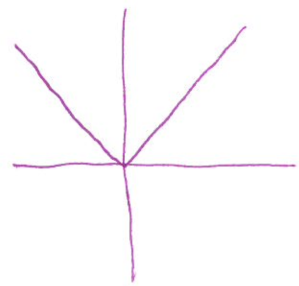
Continuous



Discontinuous

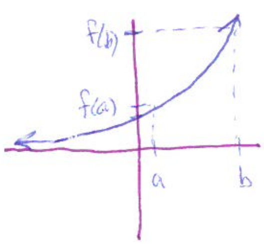


Discontinuous

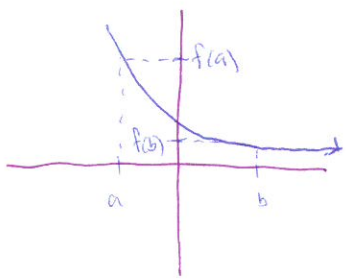


Continuous

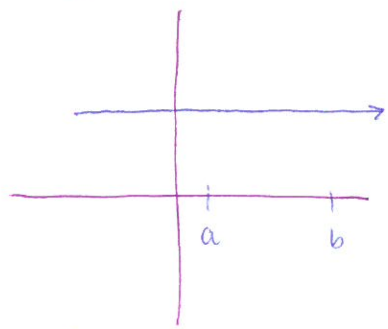
Increasing, decreasing, Constant on an Interval: [Left to right]



f is increasing if $f(a) < f(b)$ for $a < b$



f is decreasing if $f(a) > f(b)$ for $a < b$



f is constant if $f(a) = f(b)$

f has a local maximum on $[a,b]$ at $x=c$ if f goes from increasing to decreasing at $x=c$.

f has a local minimum on $[a,b]$ at $x=c$ if f goes from decreasing to increasing at $x=c$.

Continuous: Every where exact at $x=2$

Increasing: $(-\infty,-3) \cup (2,3)$

Decreasing: $(-3,2)$

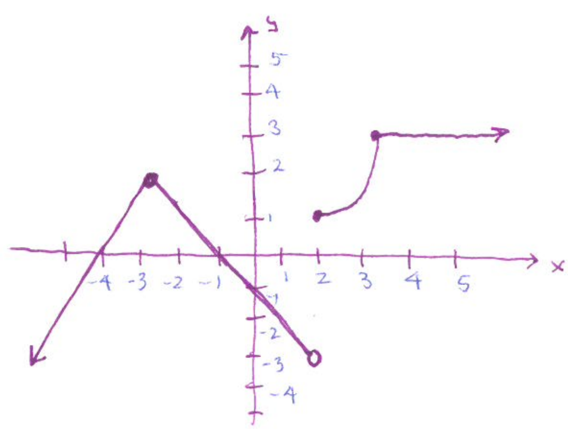
Constant: $(3,\infty)$

max: $(-3,2)$

Local min: None

Domain: $(-\infty,\infty)$ or $\mathbb{R}$

Range: $(-\infty, 3]$



Pre-Calc AB

$\boxed{\text{Functions}}$

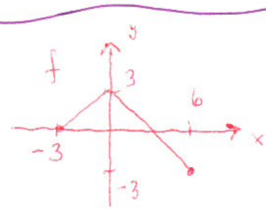
4/4/16

Topic: Functions
- transformations,
- One-to-One / horizontal line test
- Inverse functions

★ Handout Function WS II

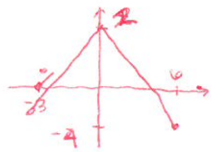
★ Quiz 13 on Wednesday
Exp. Functions
Function review

Transformations: (Review)

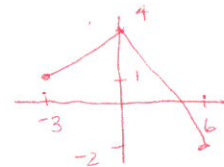


① Vertical Shifts:

$f(x)-1$ $\boxed{\text{Down}}$

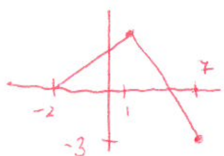


$f(x)+1$ $\boxed{\text{up}}$

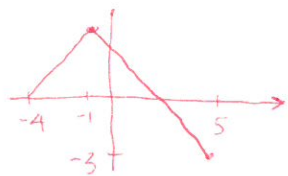


② Horizontal Shifts:

$f(x-1)$ $\boxed{\text{Right}}$

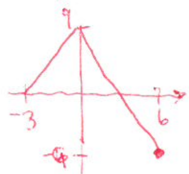


$f(x+1)$ $\boxed{\text{Left}}$

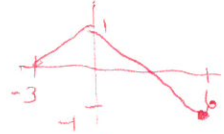


③ Vertical Shrink/stretch:

$3f(x)$ $\boxed{\text{Stretch}}$

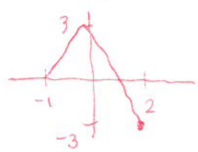


$\frac{1}{3}f(x)$ $\boxed{\text{Shrink}}$

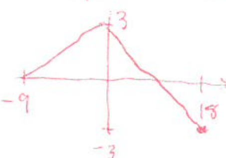


④ Horizontal Shrink/stretch:

$f(3x)$ $\boxed{\text{Shrink}}$

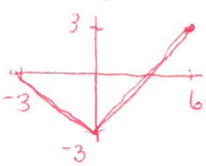


$f(\frac{1}{3}x)$ $\boxed{\text{Stretch}}$

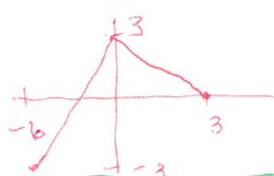


⑤ Reflections:

$-f(x)$ $\boxed{\text{x-axis}}$



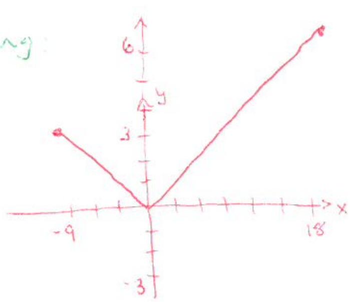
$f(-x)$ $\boxed{\text{y-axis}}$



Ex: Sketch the following:

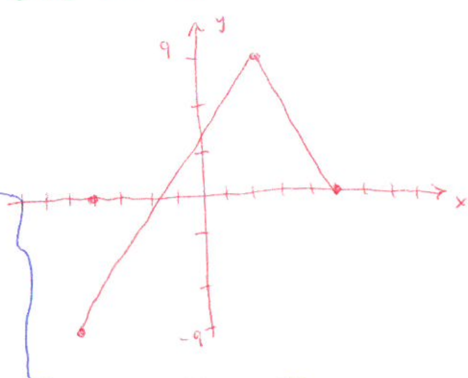
(a) $3-f(\frac{1}{3}x)$

1. HS by 3
2. Reflect x-axis
3. up 3



(b) $\cancel{+}3f(-x+2) = 3f(-(x-2))$

1. VS by 3
2. reflect y-axis
3. shift right 2

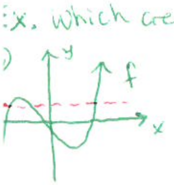


Def - A function is <u>one-to-one</u> if each output has exactly one input.
If a function is one-to-one then it has an inverse function:

$$y=f(x) \iff x=f^{-1}(y)$$

Ex. Which are one-to-one? If so find inverse:

(a)



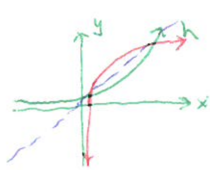
$\boxed{\text{No}}$ fails horizontal line test.

(b) $g(x) = \frac{1}{2x+3}$ $\qquad g^{-1}(x) = \frac{1}{2x} - \frac{3}{2}$

$\boxed{\text{Yes}}$ $x = \frac{1}{2y+3}$ $\qquad = \frac{1-3x}{2x}$

$2y = \frac{1}{x} - 3$

(c) $\boxed{\text{Yes}}$



Pre-Calc AB — Functions — 4/5/16

Topic: Functions

- Word problems
- As a function of

★ Handout WS 3 for functions

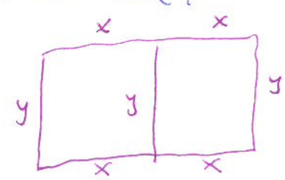
Example: A piece of cardboard is 2.5 times as long as it is wide. It is to be used to make a box with an open top by cutting 3 in squares from each corner and folding up the sides.

(a) Let x represent the width of the original piece of cardboard. Determine a function for the Volume V in terms of x.

$$V = w \cdot l \cdot 3 = \frac{5}{2}(x-6)^2 \cdot 3 = \boxed{\frac{15}{2}(x-6)^2}$$

(b) What are the restrictions on x? $\boxed{x \geq 6}$

Example: A farmer has 200 m of fence, and wishes to enclose two equal Areas righ next to each other. What are the dimensions of each Area that maximize the area?



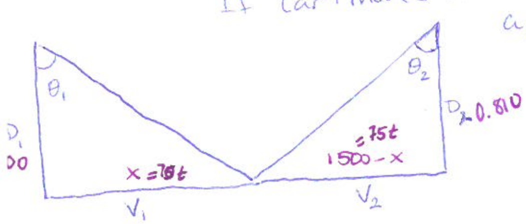
$$200 = 4x + 3y \qquad y = \frac{200-4x}{3}$$

$$A(x) = x \cdot y = x\left(\frac{200-4x}{3}\right) = \frac{200x - 4x^2}{3}$$

$$= -\frac{4}{3}\left(x^2 - \frac{50}{3}x + \frac{625}{9}\right) + \frac{2500}{27} = -\frac{4}{3}\left(x - \frac{25}{3}\right)^2 + \frac{2500}{27}$$

Max Area when $x = \frac{25}{3}$ m and $y = \frac{500}{9}$ m

Example: Camera 1 is 0.100 mi away from Car 1 and Camera 2 is 0.810 mi away from Car 2. If Car 1 moves at 70 mph and Car 2 moves at 75 mph towards each other 0.2500 miles apart.



(a) Find the angles for camera 1 and 2 as functions of time t in hours.

$$\theta_1 = \arctan\left(\frac{70t}{0.100}\right) \qquad \theta_2 = \arctan\left(\frac{75t}{0.810}\right)$$

(b) When will the two cars collide? What is θ_1, θ_2 at this time?

$$V_1 + V_2 = .2500 = 70t + 75t \qquad \text{So } t = \frac{.2500}{145} = \boxed{\frac{1}{580}\text{ hr} \approx 6.21\text{ sec}}$$

$$\theta_1 = \arctan\left(\frac{70.}{58}\right) \approx \boxed{50.36^\circ} \qquad \theta_2 = \arctan\left(\frac{75.}{52.2}\right) \approx \boxed{55.16^\circ}$$

(c) Will the distance of the camera to each car be equal before the cars collide? If so after how many seconds

$\boxed{\text{Yes}}$

$$.100^2 + (70t)^2 = (75t)^2 + 0.09^2$$

$$t \approx 0.00161885 \text{ hr} \approx \boxed{5.76\text{ sec}}$$