Pre-Calc AB

Exponential Functions

3/24/16 – 3/25/16

Topics: Exponential Functions
- Solving equations w/ calc
- word problems
- Exponential models

Examples:

- $8(4^{6-2x}) + 13 = 41$

$$4^{6-2x} = 4^1$$

$$6 - 2x = 1$$

$$\boxed{x = \frac{5}{2}}$$

- $e^{2x} + 9e^{x} - 36 = 0$

$$(e^x + 12)(e^x - 3) = 0$$

$e^x = -12$ No Solution $\qquad e^x = 3$ $\boxed{x = \ln(3)}$

- $\dfrac{400}{1 + e^{-x}} = 350$

$$\frac{40}{35} - 1 = e^{-x}$$

$$\boxed{x = -\ln\left(\frac{1}{7}\right)}$$

- $\left(4 - \dfrac{2.471}{40}\right)^{9t} = 21$

$$\boxed{t = \frac{\ln(21)}{9\ln\left(4 - \frac{2.471}{40}\right)}}$$

- **Example:** The yield V (~~millions~~ thousands of cubic meters per acre) for a corn field at t years is given by

$$V = 3.2e^{-24/t} \quad \text{for } t > 0$$

(a) Graph with Graphing calc

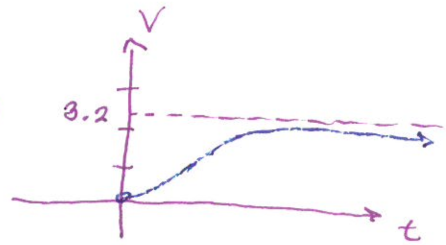
(b) Determine horizontal asymptotes and interpret

(c) find time to obtain a yield of 1.1 ~~million~~ thousand cubic meters.

$$1.1 = 3.2e^{-24/t}$$

$$-24/\ln\left(\frac{1.1}{3.2}\right) = t \approx 22.4753 \text{ years}$$

(a)



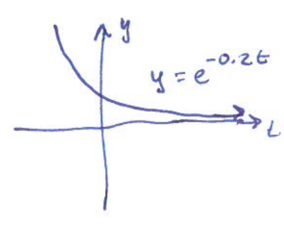
(b) $V = 3.2$

As time goes on the yield of the corn field will approach 3.2 thousand cubic meters.

Example: An animal preserve has the carrying capacity of 5000 Animals in which 200 endangered animals are released. The growth of species is modeled by

$$P(t) = \frac{5000}{1 + 199e^{-0.2t}}$$

Find and interpret the horizontal asymptote. As $t \to \infty$ where does $P(t)$ go



$$\lim_{t\to\infty} \frac{5000}{1 + 199e^{-0.2t}} = \frac{5000}{1 + 199(0)} = 5000 \quad \text{since} \quad \lim_{t\to\infty} e^{-0.2} = 0$$

As time goes on the population will reach 5000.

Example: In a research experiment, a population of fruit flies increases exponentially. After 2 days there are 100 flies and after 4 days there are 300 flies. How many flies will there be after 5 days?

Model: $F(t) = F_0 e^{kt}$ Find F_0 and k

① $100 = F_0 e^{2k}$ ② $300 = F_0 e^{4k}$

$$3 = e^{2k} \Rightarrow k = \boxed{\tfrac{1}{2}\ln(3)}$$

$$F_0 = \frac{100}{e^{2k}} = \frac{100}{e^{\ln(3)}} = \boxed{\frac{100}{3}}$$

Model: $F(t) = \frac{100}{3} e^{\frac{1}{2}\ln(3)t}$

After 5 days there will be $\frac{100}{3} e^{\frac{1}{2}\ln(3)\cdot 5} \approx \boxed{519 \text{ fruit flies}}$

Example On a college campus of 5000 students, one student returns from vacation with a contagious and long-lasting flu virus. The spread is modeled by

$$y = \frac{5000}{1 + 4999e^{-0.8t}} \quad \text{for } t \geq 0$$

where y is the total number of students infected after t days. The college will cancel classes when 40% or more of the students are infected. ~~[illegible]~~ After how many days will the college cancel classes?

$$0.4(5000) = \frac{5000}{1 + 4999e^{-0.8t}}$$

$$4999e^{-0.8t} = \frac{3}{2}$$

$$t = -\frac{1}{0.8}\ln\left(\frac{1.5}{4999}\right) \approx \boxed{10.1 \text{ days}}$$

Example A Dell Inspiron 8600 laptop costs \$1150 new and has a book value of \$550 after 2 years.

(a) Find a linear model $V = mt + b$

(b) Find an exp. model $V = ae^{kt}$

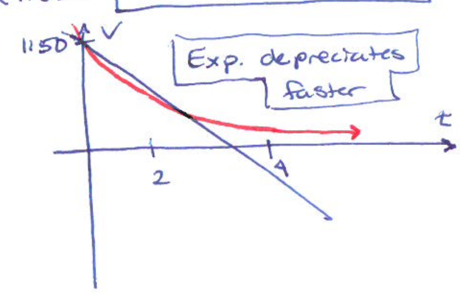
(c) Which depreciates faster in the first 2 years?

(d) Find the book value after 3 years using both models.

$550 = m(2) + 1150$

$m = -300$ $\boxed{V = -300t + 1150}$

$k = \frac{1}{2}\ln\left(\frac{550}{1150}\right)$ $\boxed{V = 1150e^{\frac{1}{2}\ln\left(\frac{55}{115}\right)t}}$



Linear	$-300(3) + 1150 = \$250$
Exp	$1150e^{\frac{1}{2}\ln\left(\frac{55}{115}\right)3} \approx \$380.36$