Pre-Calc AB

Inverse Trig Functions

3/21/16
– 3/22/16

Topics: Inverse Trig Functions (110)

- arcsine, arccosine
- arctan, arccot
- arcsec, arccsc
- Domain and range
- inverse properties
- Applications and models

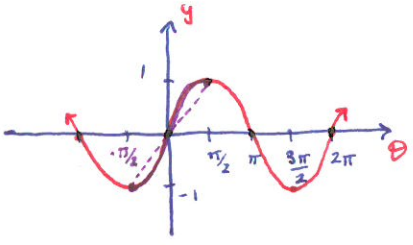
★ Handout Inverse Trig WS I
★ Handout Test Study Guide
★ Take home quiz due tomorrow

Inverse Function: $f^{-1}(x)$ is the inverse of $f(x)$ if

$$f(x) = y \iff f^{-1}(y) = x$$

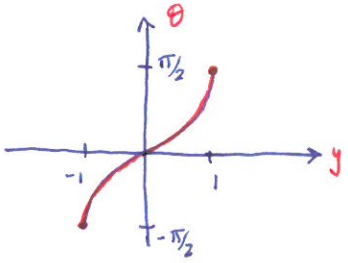
One-to-One: Every output has exactly one input

$$f(x) = f(y) \Rightarrow x = y$$



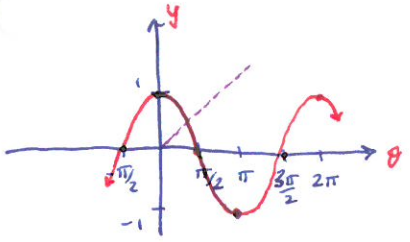
$y = \sin\theta$

★ Only 1-1 on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



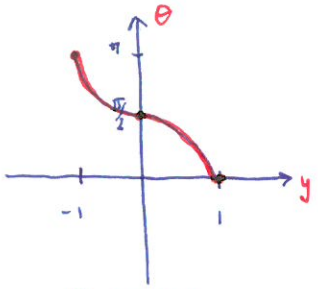
$\theta = \arcsin y$

Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



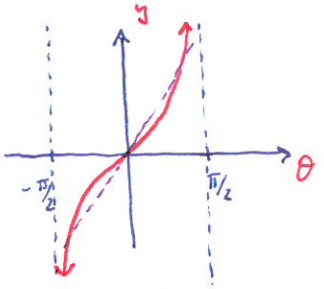
$y = \cos\theta$

★ Only 1-1 on the interval $[0, \pi]$



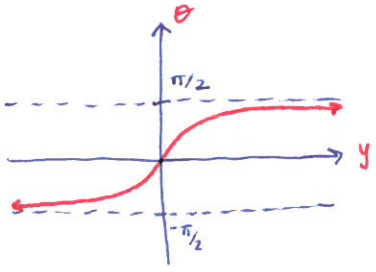
$\theta = \arccos y$

Domain: $[-1, 1]$
Range: $[0, \pi]$



$y = \tan\theta$

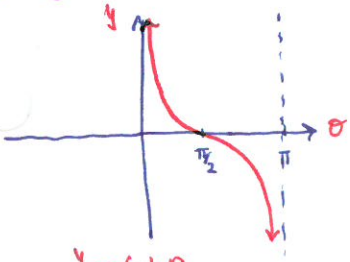
★ Only 1-1 on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$\theta = \arctan y$

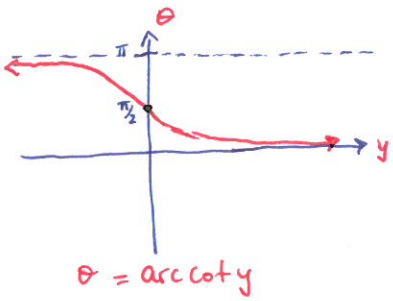
Domain: $\mathbb{R}$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

★ Asymptotes where $\cos\theta = 0$



$y = \cot\theta$

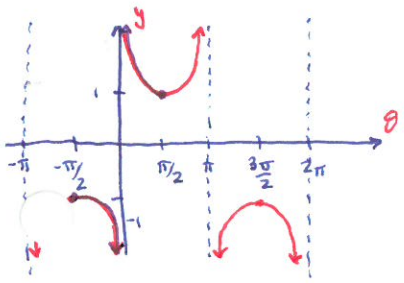
★ Only 1-1 on the interval $[0, \pi]$



$\theta = \operatorname{arccot} y$

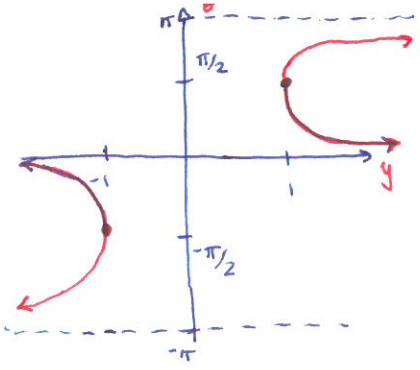
Domain: $\mathbb{R}$
Range: $[0, \pi]$

★ Asymptotes where $\sin\theta = 0$



$y = \csc\theta$

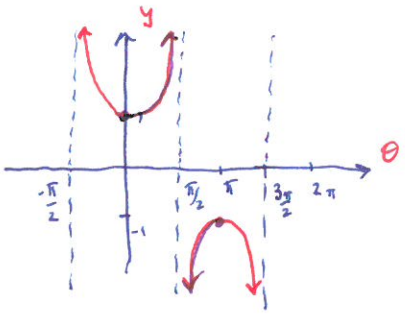
★ 1-1 on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$



$\theta = \operatorname{arccsc} y$

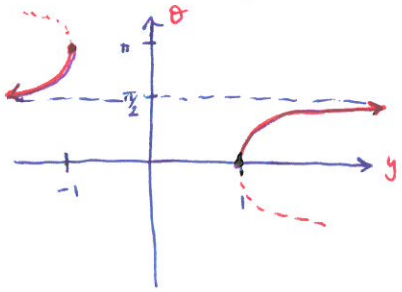
Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



$y = \sec\theta$

★ 1-1 on the interval $[0, \pi/2) \cup (\pi/2, \pi]$



$\theta = \operatorname{arcsec} y$

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \pi/2) \cup (\pi/2, \pi]$

Inverse Properties: ① If $-1 \le y \le 1$ and $-\pi/2 \le \theta \le \pi/2$ then

$\sin(\arcsin y) = y$ and $\arcsin(\sin\theta) = \theta$

② If $-1 \le y \le 1$ and $0 \le \theta \le \pi$ then

$\cos(\arccos y) = y$ and $\arccos(\cos\theta) = \theta$

③ If y is a real number and $-\pi/2 < \theta < \pi/2$ then

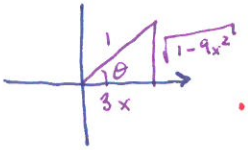
$\tan(\arctan y) = y$ and $\arctan(\tan\theta) = \theta$

Examples:

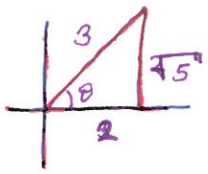
- $\tan(\arctan(-5)) = \boxed{-5}$
- $\arcsin\left(\sin\left(\frac{5\pi}{8}\right)\right) = \arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right) = \boxed{-\frac{\pi}{3}}$
- $\cos(\arccos(\pi)) = \boxed{\text{undefined}}$ $\quad \pi \notin [-1, 1]$
- $\sin(\arccos 3x)$ for $0 \le x \le \frac{1}{3}$

  $= \sin\theta$

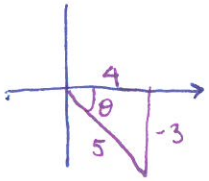
  $= \boxed{\sqrt{1-9x^2} \text{ for } 0 \le x \le \frac{1}{3}}$



- $\tan\left(\arccos\frac{2}{3}\right) = \tan\theta = \boxed{\frac{\sqrt{5}}{2}}$



- $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right) = \cos\theta = \boxed{\frac{4}{5}}$



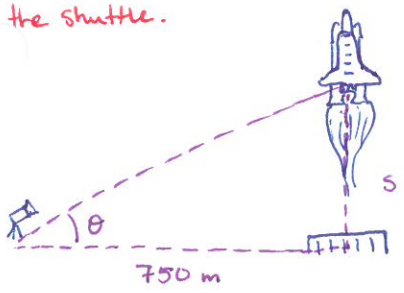
- $\cot(\arccos 3x)$ for $0 \le x \le \frac{1}{3}$

  $= \cot\theta$

  $= \boxed{\frac{3x}{\sqrt{1-9x^2}} \text{ for } 0 \le x \le \frac{1}{3}}$

Ex. (92)

A television camera at ground level is filming the lift-off of a space shuttle at a point 750m from the launch pad. Let θ be the angle of elevation to the shuttle and let s be the height of the shuttle.



(a) Write θ as a function of s.

$$\tan\theta = \frac{s}{750} \Rightarrow \boxed{\theta = \arctan\left(\frac{s}{750}\right)}$$

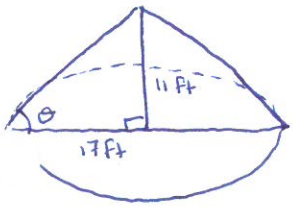
(b) Find θ when $s = 300$ m and $s = 1200$ m

$$\theta(300) = \arctan\left(\frac{300}{750}\right) \approx \boxed{21.8^\circ}$$

$$\theta(1200) = \arctan\left(\frac{1200}{750}\right) \approx \boxed{57.99^\circ}$$

Ex. (94)

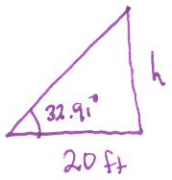
Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle θ is called the angle of repose. When rock salt is stored in a cone-shaped pile 11 ft high, diameter is 34 ft.



(a) Find the angle of repose for rock salt.

$$\tan\theta = \frac{11}{17} \quad \text{so} \quad \theta = \arctan\left(\frac{11}{17}\right) \approx \boxed{32.91^\circ}$$

(b) How tall is a pile of rock salt that has a base diameter of 40 ft?



$$h = 20\tan 32.91^\circ = \frac{20\cdot 11}{17} \approx \boxed{12.94 \text{ ft}}$$