Pre-Calc AB

Rational & Prime Numbers

3/7/16

Topics: lessons 109, 117
- rational numbers
- prime numbers
- relatively prime
- Proof that $0.\overline{9} = 1$

- Handout Rational & Primes WS
- Quiz 12 on Wednesday

Definition - a rational number is any number that can be written as a ratio of two integers

Notation: $\mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z},\ b \neq 0 \right\}$

Ex. $\frac{4}{6}$, $1\frac{1}{2}$, 0.23, -3 are all examples of rational numbers

$$\frac{4}{6},\ \frac{3}{2},\ \frac{23}{100},\ \frac{-3}{1}$$

Ex. Use an infinite geometric series to write $0.00\overline{23}$ as a ratio of two integers.

$$0.00\overline{23} = 0.0023 + 0.000023 + 0.00000023 + \cdots$$

$a_1 = 0.0023 = 23 \times 10^{-4}$

$r = 10^{-2}$

$$= 0.0023 \sum_{i=0}^{\infty} (10^{-2})^i = \frac{0.0023}{1-(10^{-2})} = \frac{0.0023}{0.99} = \frac{\frac{23}{10000}}{\frac{99}{100}} = \frac{23 \cdot 100}{10000 \cdot 99} = \boxed{\frac{23}{9900}}$$

Ex. Prove that $0.\overline{9} = 1$

$$0.\overline{9} = 0.9 + 0.09 + 0.009 + \cdots$$

$a_1 = 0.9$

$r = 10^{-1}$

$$= a_1 \sum_{i=0}^{\infty} r^i = \frac{a_1}{1-r} = \frac{9 \times 10^{-1}}{90 \times 10^{-2}} = \frac{90 \times 10^{-2}}{90 \times 10^{-2}} = \boxed{1} \quad ∎$$

Definitions - A composite number is a counting number that is the product of two other counting numbers.

Counting numbers that are not composite are called prime.

Two counting numbers are relatively prime if there only common factor is 1.

Ex. Are 330 and 539 relatively prime?

$330 = 33 \cdot 10 = 3 \cdot 2 \cdot 5 \cdot 11$

$539 = 49 \cdot 11 = 7^2 \cdot 11$

No Common factor is 11.

Ex.

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Binomial Theorem

3/8/16

Topics: lessons 102, 112

- Binomial Expansion
- Combinations in Pascal's Triangle
- Binomial Theorem

- Handout Binomial WS
- Quiz Tomorrow

Ex Expand $(2a^2 - x^3)^4$

$$= (2a^2)^4 + 4(2a^2)^3(-x^3) + 6(2a^2)^2(-x^3)^2 + 4(2a^2)(-x^3)^3 + (-x^3)^4$$

$$= \boxed{16a^8 - 32a^6x^3 + 24a^4x^6 - 8a^2x^9 + x^{12}}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 $\boxed{10}$ 10 5 1

1 6 15 20 15 6 1

$\binom{n}{0}$ $\binom{n}{1}$ $\binom{n}{2}$ $\binom{n}{3}$ $\binom{n}{4}$

1 2 3 4 5 ...

Ex. Find the 3rd term in $(-3x + 2y^2)^5$

$$3^{rd} \text{ term} = 10(-3x)^3(2y^2)^{5-3}$$

$$= 10(-27x^3)(4y^4)$$

$$= \boxed{-1080x^3y^4}$$

k^{th} term in $(a+b)^n$

$$\binom{n}{k-1}(a)^{n-k+1}(b)^{k+1}$$

Binomial Theorem:

The k^{th} term in the expansion of $(a+b)^n$, where $k \le n+1$ is

$$\binom{n}{k-1}(a)^{n-k+1}(b)^{k-1}$$

That is

$$\boxed{(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i}$$

Ex. Find the eigth term of $(F+S)^8$

$n = 8$

$k = 8$

$$8^{th} \text{ term} = \binom{8}{7}(F)^{8-8+1}(S)^{8-1} = 8 \cdot F \cdot S^8$$

Ex. Find the 10th term of $(2x^3 - y)^{15}$

$n = 15$

$k = 10$

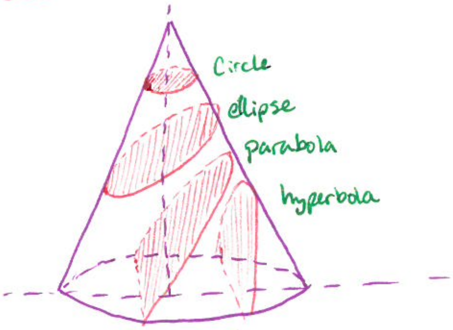
$$10^{th} \text{ term} = \binom{15}{10-1}(2x^3)^{15-10+1}(-y)^{10-1} = \frac{15!}{9!\,6!}(2x^3)^6(-y)^9 = 5005(64x^{18})(-y^9)$$

$$= \boxed{-320{,}320x^{18}y^9}$$

Pre-Calc AB | Conic Sections | 3/10/16 – 3/11/16

Topic: Conic Sections (106 & 123)

- Types of Conic sections
- Translated conic sections
- The General Conic Equation



Circle:

Center at (0,0): $x^2 + y^2 = r^2$

Center at (h,k): $(x-h)^2 + (y-k)^2 = r^2$

Ellipse:

Center at (0,0): $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Center at (h,k): $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Parabola:

Vertex at (0,0): $y = ax^2$

Vertex at (h,k): $y - k = a(x-h)^2$

hyperbola:

Center at (0,0): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Center at (h,k): $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Ex. 106.1

Write the ellipse

$3x^2 + 2y^2 - 6x + 8y + 5 = 0$

in standard form:

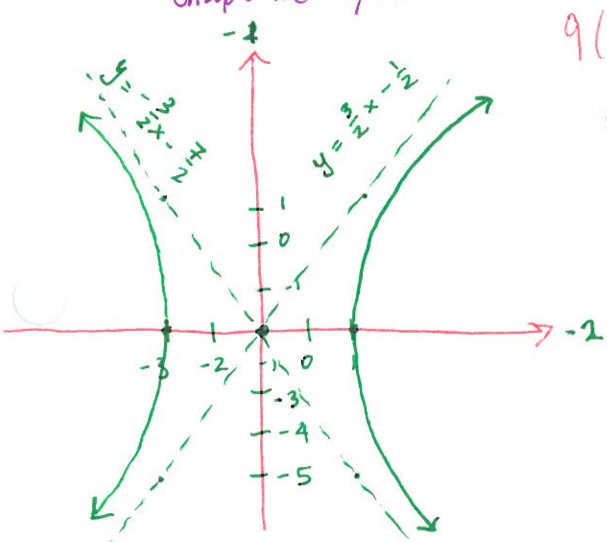
$3(x^2 - 2x + 1) + 2(y^2 + 4y + 4) = -5 + 3 + 8$

$\frac{3(x-1)^2}{6} + \frac{2(y+2)^2}{6} = \frac{6}{6}$

$$\boxed{\frac{(x-1)^2}{(\sqrt{2})^2} + \frac{(y+2)^2}{(\sqrt{3})^2} = 1}$$

Ex 106.2 Write $9x^2 - 4y^2 + 18x - 16y - 43 = 0$ in standard form to determine the conic section

Graph the equation.



$9(x^2 + 2x + 1) - 4(y^2 + 4y + 4) = 43 + 9 - 16$

$\frac{9(x+1)^2}{36} - \frac{4(y+2)^2}{36} = \frac{36}{36}$

$\frac{(x+1)^2}{(2)^2} - \frac{(y+2)^2}{3^2} = 1$

Center: (-1, -2)

Asymptotes:

$(y+2)^2 = \frac{9}{4}(x+1)^2$

$y = \pm \frac{3}{2}(x+1) - 2$

$y = \frac{3}{2}x - \frac{1}{2}$

$y = -\frac{3}{2}x - \frac{7}{2}$

$\boxed{\text{General Conic Section}}$ $ax^2 + bxy + cy^2 + dx + ey + f = 0$ 3/11/16

- $b = c = 0$, $a, e \neq 0$ Then we have a parabola

Ex. $x^2 + 4x - y + 1 = 0 \Rightarrow y = (x+2)^2 - 3$

- $b = 0$, $a = c \neq 0$ then we have a circle

$x^2 + y^2 - 8x - 4y + 11 = 0 \Rightarrow (x-4)^2 + (y-2)^2 = 9$

- If $b = 0$, $a \neq c$ but have same sign nonzero then have an ellipse

$4x^2 + 3y^2 + 4x - 2y = 0 \Rightarrow 4(x+\frac{1}{2})^2 + 3(y-$

- If $b = 0$, $a \neq c$ nonzero opposite signs then have a hyperbola

$4x^2 - 3y^2 + 4x - 3y + 7 = 0$

Degenerate Conic Sections: A point, two parallel lines, two intersecting lines, no graph

$x^2 + y^2 = 0$ $\quad (y - x)^2 = 1$ $\quad y^2 = (x)^2$ $\quad x^2 + y^2 = -1$

One line, reciprocal function

$x + y + 1 = 0$ $\quad xy + 2 = 0$

Example: Classify the following

- $9x^2 - 16y^2 + 18x + 64y - 199 = 0$

$9(x^2 + 2x + 1) - 16(y^2 - 4y + 4) = 199 + 9 - 64$

$9(x+1)^2 - 16(x-2)^2 = 144$

$\frac{(x+1)^2}{16} - \frac{(x-2)^2}{9} = 1$

$\boxed{\text{Hyperbola}}$

- $x^2 + 4y^2 + 2x - 32y + 61 = 0$

$(x+1)^2 + 4(y-4)^2 = 4$

$\frac{(x+1)^2}{4} + (y-4)^2 = 1$

$\boxed{\text{Ellipse}}$

- $x^2 + y^2 - 2x + 2y - 7 = 0$

$(x-1)^2 + (y+1)^2 = 9$

$\boxed{\text{Circle}}$

- $x^2 + 12x + y + 39 = 0$

$y = -(x+6)^2 - 3$

$\boxed{\text{Parabola}}$