Pre-Calc AB

Polynomials

4/19/16

Topic: Polynomials

- Definition
- even/odd Powers
- End behavior
- local extrema
- Number of zeros
- product of distinct linear factors

★ Handout WS Poly 1

★ Quiz 14 tomorrow (Last Quiz)

Definition - A polynomial function of degree n is a function defined by

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

Where each $a_i \in \mathbb{R}$ and $a_n \neq 0$ and n is a whole number.

★ Domain is $\mathbb{R}$

★ Always Continuous

- $a_n x^n$ - Leading term
- n - degree
- a_n - Leading coefficient
- a_0 - Constant term

Ex.

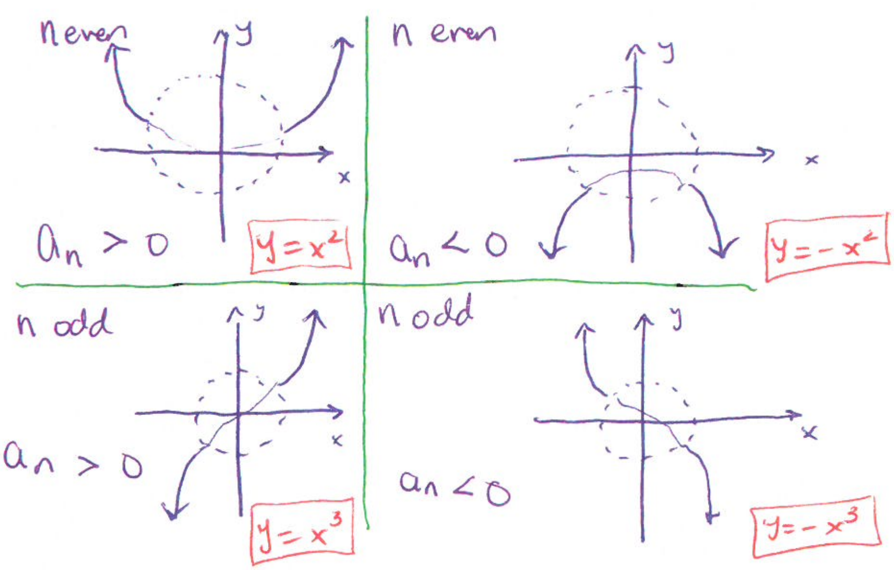
$f(x) = 2x^2 + 4$

$f(x) = \frac{1}{2}x^3 - 3x + 1$

$f(x) = 2$

$f(x) = \sqrt{x-1}$, $f(x) = \dfrac{1}{x-1}$ } Not polynomials

End Behavior:



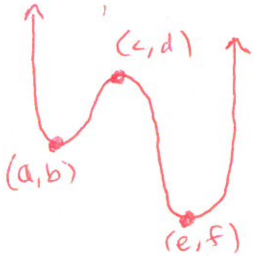
Extrema

Local Extrema and Absolute Extrema (minimas and maximas)

Let c be in the domain of P

- $P(c)$ is an absolute max if $P(c) \geq P(x)$ for all x in the domain.
- $P(c)$ is an absolute min if $P(c) \leq P(x)$ for all x in the domain.
- $P(c)$ is a local max if $P(c) \geq P(x)$ for all x in an open interval containing c.
- $P(c)$ is a local min if $P(c) \leq P(x)$ for all x in an open interval containing c.

Ex.



Local max: (c, d)

local min: (a,b) (e,f)

Abs. max: None

Abs. min: (e, f) ← Abs. min value

Note:
A function may have more than one absolute min/max points but can only have one abs max/min value.

Number of Turning points (Local Extrema)

of the graph of a polynomial of degree $n \geq 1$ is at most $n-1$.

Number of x-intercepts

of a polynomial of degree n will have at most n x-intercepts (real zeros).

Graphing with Distinct Linear factors:

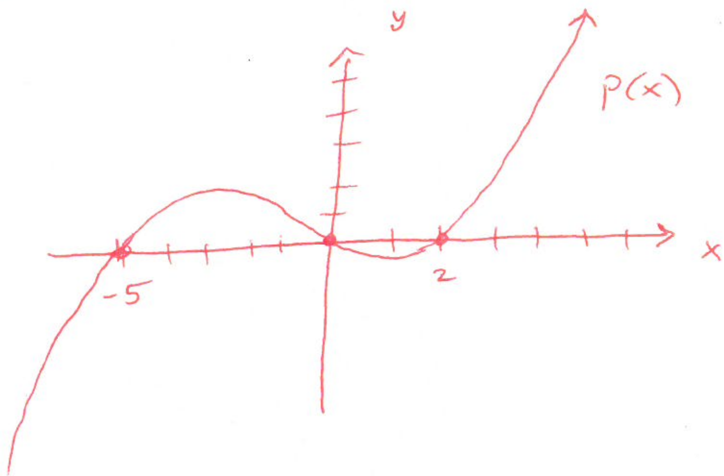
Ex $P(x) = (x-2)(x+5)(x)$

Degree: 3

End behavior:

Zeros: 2, −5, 0

y-intercept: (0, 0)



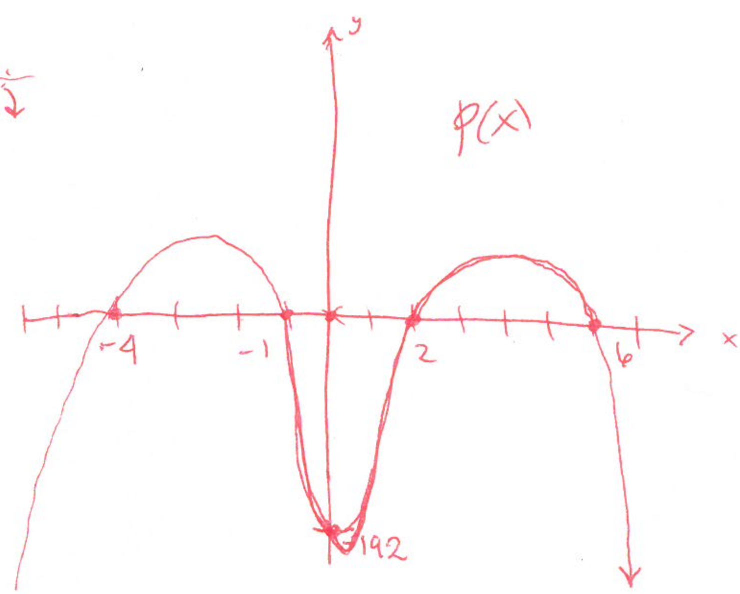
Ex. $P(x) = +2(-2x+4)(x+4)(x-6)(x+1)$

Degree: 4

End behavior: $(2(-2)<0)$

Zeros: $x = 2, -4, 6, -1$

y-interept: $(0, -192)$



Pre-Calc AB | $\boxed{\text{Polynomials}}$ | 4/21/16

Topic: Polynomials

- Graphs
- irreducible factors
- factors with multiplicity > 1

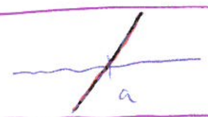
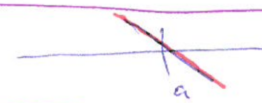
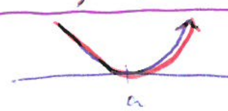
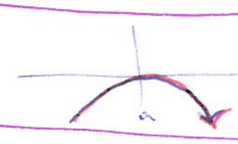
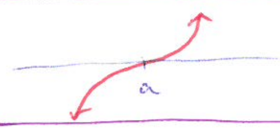
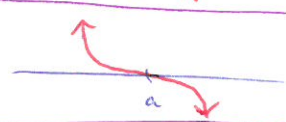
★ Handout WS Poly 2

★ Quiz back after lesson

Definition - An irreducible factor of a polynomial is a factor that has no real roots or never causes a polynomial to be 0.

The multiplicity of a root (zero) is the number of times the factor occurs for the root in the polynomial.

What the Graph Looks like at a root: $(x-a)^r$

multiplicity			
$r = 1$		or	
r is even		or	
r is odd		or	

Ex. Find all zeros of $f(x) = (x+4)(x+2)^4(x^2+3)(x-5)^3(x-7)^2$ and state which touch and which cross the x-axis.

Zeros (touch): $x = -2, 7$

Zeros (cross): $x = -4, 5$

x^2+3 is an irreducible factor

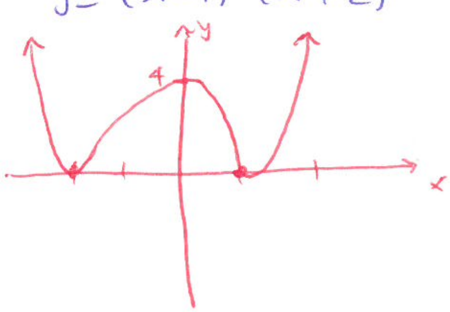
Ex. Sketch $y = (x-1)^2(x+2)^2$

Degree: 4

End: ↑ ↗

Zeros (touch): $x = 1, -2$

y-intercept: $(0, 4)$



$y = -x^3(x^2+4)(x+5)^2(x-3)^4$

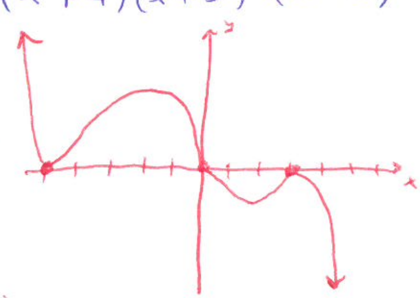
Degree: 11

End: ↖ ↘

Zeros (touch) $x = -5, 3$

Zeros (cross) $x = 0$

y-intercept $(0, 0)$



Pre-Calc AB

Polynomials

4/22/16

Topic: Polynomials
- Factoring polynomials
- Finding local extrema with calc
- Finding zeros on calc

★ Handout WS Poly 3

Remainder Theorem: $\frac{P(x)}{x-a} = P(a)$ in particular $x-a$ is a factor of $P(x)$ iff $P(a)=0$.

Long Division:

Ex. Check if $x=-3$ is a root of $y = 2x^4 - 32x$

$$\begin{array}{r} 2x^3 - 6x^2 + 18x - 86 \\ x+3 \,\overline{)\, 2x^4 + 0x^3 + 0x^2 - 32x + 0} \\ -(2x^4 + 6x^3) \\ \hline -6x^3 + 0x^2 \\ -(-6x^3 - 18x^2) \\ \hline 18x^2 - 32x \\ -(18x^2 + 54x) \\ \hline -86x + 0 \\ -(-86x - 258) \\ \hline +258 \end{array}$$

$y(-3) = 2(-3)^4 - 32(-3) = 258$

So

$$\frac{2x^4 - 32x}{x+3} = 2x^3 - 6x^2 + 18x - 86 + \frac{258}{x+3}$$

$\boxed{\text{So not a root}}$

Ex. Check if $x=-1$ is a zero of $x^3 + 2x^2 - 3x - 4$

$$\begin{array}{r} x^2 + x - 4 \\ x+1 \,\overline{)\, x^3 + 2x^2 - 3x - 4} \\ -(x^3 + x^2) \\ \hline x^2 - 3x \\ -(x^2 + x) \\ \hline -4x - 4 \\ -(-4x - 4) \\ \hline 0 \end{array}$$

Yes $x^3 + 2x^2 - 3x - 4 = \boxed{(x+1)(x^2+x-4)}$

$(-1)^3 + 2(-1)^2 - 3(-1) - 4 = -1 + 2 + 3 - 4 = 0$

Rational Roots Theorem: If an equation $a_n x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0 = 0$ has a rational root then it must be of the form:

$$x = \frac{\text{factor of } a_0}{\text{factor of } a_n}$$

Example: What are the possible rational roots of $y = 3x^4 - 9x^2 - 5x - 2$

$$x = \frac{\{2, 1, -2, -1\}}{\{3, 1, -1, -3\}} = \boxed{\tfrac{2}{3}, \tfrac{1}{3}, -\tfrac{2}{3}, -\tfrac{1}{3}, 2, 1, -2, -1}$$

X X X X X X X

~~Example~~: Find the zeros and local extrema of $y = x^5 - 8x^3 + x^2 + 1$

Zeros: $x \approx -1.3523, 0.60494, 1.13430$

local max: $(-1.01907, 5.00968)$ local min: $(0.93512, -1.09202)$