Pre-Calc AB

Series

2/29/16 - 3/1/16

Series Topics

- finite, infinite
- Arithmetic, Geometric
- Formula for n^{th} sum, infinite sum
- Convergent Geometric series

★ 3/1 Handout Series WS

★ Lessons 104 + 107

A Series is the indicated sum of a sequence.

A finite series is the sum of a finite sequence. An infinite series is the sum of an infinite sequence.

An Arithmetic Series is the sum of an arithmetic sequence.

A Geometric series is the sum of an Geometric sequence.

Ex. Series for the arithmetic sequence whose first term is a_1 and common diff is d. and whose last term is a_n

$$\text{Series} = a_1 + a_2 + a_3 + a_4 + \cdots + a_n = \boxed{\frac{n}{2}(a_1 + a_n) = \frac{n}{2}(2a_1 + d(n-1))}$$

$$= a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \cdots + a_1 + (d(n-1))$$

$$= a_1 \cdot n + d(1 + 2 + 3 + 4 + \cdots + n-1)$$

$$= \boxed{a_1 \cdot n + d \sum_{i=1}^{n-1} i}$$

$a_1 \; a_2 \; a_3 \cdots a_{\frac{n}{2}} \cdots a_{n-2} \; a_{n-1} \; a_n$

$a_1 + a_n = a_1 + a_1 + d(n-1)$ //

$(a_1 + d) + a_1 + d(n-2)$

$\vdots$

For infinite/finite series we can define partial sums:

For sequence $\{a_n\}$ the n^{th} partial sum: $\boxed{S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n}$

Ex. 104.1 $a_1 = -10$, $d = 20$ Find the 11th partial sum:

$$S_{11} = a_1 + \cdots + a_{11} = \frac{n}{2}(a_1 + a_{11}) = \frac{11}{2}\left(-10 + (-10 + 10(20))\right) = \boxed{990}$$

Ex. $a_2 = x$, $a_7 = 13x$ Find the 13th partial sum:

$$x = a_1 + d(2-1) \qquad 13x = a_1 + d(6) \qquad S_{13} = \frac{13}{2}(-5x + 31x) = \frac{13}{2} \cdot 26x = \boxed{169x}$$

$$12x = 4d$$

$$d = \frac{12x}{4} = 3x \implies a_1 = -5x$$

$$a_{13} = -5x + 3x(12) = -5x + 36x = 31x$$

Ex. Series for geometric sequence with first term a_1, common ratio r, last term a_n [or the nth partial sum]

$$S_n = a_1 + a_2 + a_3 + a_4 + \cdots + a_n = \sum_{i=1}^{n} a_i =$$

$$= a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} = a_1 \sum_{i=0}^{n-1} r^i = a_i \sum_{i=1}^{n} r^{i-1}$$

$$- a_1 r - a_1 r^2 - \cdots - a_1 r^{n-1} - a_1 r^n$$

$$S_n - r S_n = a_1 - a_1 r^n$$

$$\boxed{S_n = \frac{a_1 (1 - r^n)}{1 - r}} \boxed{= a_1 \sum_{i=0}^{n-1} r^i}$$

Ex. Find the 9th partial sum of a geometric sequence with $a_1 = -8$ and $r = -2$.
Write an equation for the nth partial sum.

$$S_9 = \frac{-8(1 - (-2)^9)}{1 - (-2)} = \frac{-8}{3}(1 + 512) = \boxed{-1368}$$

$$\boxed{S_n = \frac{-8(1 - (-2)^n)}{3}}$$

Find the 10th partial sum: $S_{10} = -\frac{8}{3}(1 - (-2)^{10}) = \boxed{2728}$

For infinite series we say it

convergent - if the sum is a finite number

divergent - if the sum is infinite

For a Geometric Series

$$\text{nth partial sum } \boxed{S_n = a_1 \sum_{i=0}^{n-1} r^i = \frac{a_1(1-r^n)}{1-r}, \ \underbrace{S = a_1 \sum_{i=0}^{\infty} r^i}_{\text{infinite geo series}}}$$

We say S is the limit of S_n as n goes to ∞

Write $\lim_{n \to \infty} S_n = S$

★ Now as $n \to \infty$ $r^n \to \pm\infty$ if $|r| > 1$, 1 if $r = 1$ and 0 if $|r| < 1$

Therefore $S = a_1 \sum_{i=0}^{\infty} r^i$ is convergent if $|r| < 1$ and divergent if $|r| \geq 1$

Since $\lim_{n \to \infty} r^n = 0$ for $|r| < 1$ $\Rightarrow$ $S = a_1 \sum_{i=0}^{\infty} r^i = \boxed{a_1 \cdot \left(\frac{1}{1-r}\right)}$

Ex. 107.1

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Find the sum of the geometric series: $5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \ldots$

$a_1 = 5$ and $r = \frac{1}{2}$ $\quad a_1 \sum_{i=0}^{\infty} r^i = \frac{a_1}{1-r} = \frac{5}{1-\frac{1}{2}} = \boxed{10}$

Ex. 107.2

A ball is dropped from a height of 12 feet and rebounds 2/5 of the fall distance on each succeeding bounce.

(a) How far will the ball fall on the tenth fall?

(b) What will be the total distance the ball travels?

(a) $a_1 = 12 \quad r = \frac{2}{5} \qquad a_{10} = a_1 r^9 = 12 \cdot \left(\frac{2}{5}\right)^9 \approx \boxed{0.0031452 \text{ ft}}$

(b) Total Distance

$$= 12 + 12 \cdot 2 \sum_{i=1}^{\infty} \left(\frac{2}{5}\right)^i = 12 + 24\left(\frac{a_1}{1-r}\right) = 12 + 24\left(\frac{2/5}{1 - 2/5}\right)$$

$$= 12 + 24\left(\frac{2}{3}\right)$$

$$= \boxed{28 \text{ ft}}$$