Pre-Calc AB

Lesson 96 + 97

2/19/16

Agenda: 2/19/16

Lesson 96 + 97

- Double Angle identies (2)] (96)
- Triangle Area formula] (96)
- The Ambiguous Case] (97)

Ex. 96.1 Show: $(\sin x + \cos x)^2 = 1 + \sin 2x$

$LHS = \sin^2 x + 2\sin x \cos x + \cos^2 x$ [Distribute]

$= 1 + 2\sin x \cos x$ [pythagorean identity]

$= 1 + \sin 2x$ [Double Angle identity]

$= RHS$ ∎

Ex 96.2 Show: $\dfrac{\cos^4 x - \sin^4 x}{\cos 2x} = 1$

$LHS = \dfrac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\cos 2x}$ [Difference of squares]

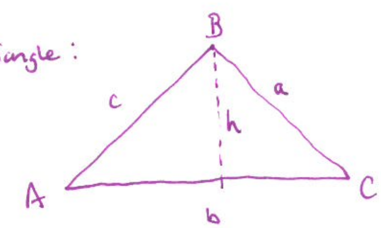
$= \dfrac{\cos^2 x - \sin^2 x}{\cos 2x}$ [pythagorean identity]

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x}$ [Double Angle identity]

[Canceling]

$= 1 = RHS$ ∎

Area of any triangle:

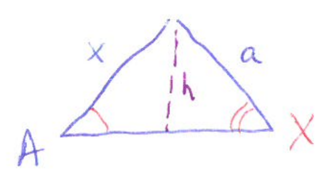


$\text{Area} = \frac{1}{2} b \cdot h = \boxed{\frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C}$

$h = c \sin A$ or $h = a \sin C$

The Ambiguous Case: Law of Sines

★ Ambiguity when One pair and one other side is known:



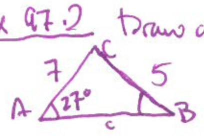
$\sin X = \dfrac{x \cdot \sin A}{a}$

1. No Solution if $\dfrac{x \sin A}{a} > 1$

2. One Solution if $A \geq 90°$ $h, x < a$
or if one of $\begin{cases} X = \arcsin\left(\frac{x \sin A}{a}\right) \text{ or} \\ X = 180° - \arcsin\left(\frac{x \sin A}{a}\right) \end{cases}$ Don't work

3. $X = \arcsin\left(\dfrac{x \sin A}{a}\right)$ and $X = 180° - \arcsin\left(\dfrac{x \sin A}{a}\right)$ $h = x \sin A < a < x$

Ex 97.2 Draw and find all missing angles: $A = 27°$, $a = 5$, $b = 7$



$A < B$

$5 < 7$

$\sin B = \dfrac{7 \cdot \sin 27°}{5} \Rightarrow B \approx 39.46°$ or $B = 140.54°$

$B \approx 39.46° \rightarrow C = 113.54°$ $c = 10.10$

$B = 140.54° \longrightarrow C = 12.46°$ $c = 2.38$

$h = 7 \sin 27° \approx 3.18 < 5 < 7 \Rightarrow 2$ Solutions

Agenda: 2/22/16
lesson 98
Change of Base
Contrived log Problems

★ Rhodeo Break 2/24 - 2/28

Ex. Find $\log_7 9 = y$

$7^y = 9 \Rightarrow \log(7^y) = \log(9)$, $\ln(7^y) = \ln(9) \Rightarrow y = \dfrac{\ln(9)}{\ln(7)} \approx 1.12915$

Change of Base Formula:

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)} \quad \text{or} \quad \log_b(a) = \frac{\ln(a)}{\ln(b)}$$

Contrived log problems

Ex. 98.3 Solve: $\log(x^2) = (\log x)^2$

$2\log x = (\log x)^2$

$(\log x)^2 - 2\log x = 0$

$\log x(\log x - 2) = 0$

$\log x = 0$ or $\log x = 2$

$\boxed{x = 1}$ or $\boxed{x = 100}$

Ex. Solve $\log_7(\log_7(x)) = 3$

$\cancel{7^{\log_7}}(\log_7(x)) = 7^3$

$\log_7 x = 7^3$

$\boxed{x = 7^{(7^3)}}$

Ex. 98.6 Solve $x^{\sqrt{\log x}} = e^8$

$\log_e\left(x^{\sqrt{\log_e x}}\right) = 8$

$\sqrt{\log_e x} \cdot \log_e x = 8$

$(\log_e x)^{3/2} = 8$

$\log_e x = 8^{2/3}$

$\log_e x = 4$

$\boxed{x = e^4}$

Agenda: 2/23/16
lesson 99
- Sequence Notation
- Advanced Sequence problems
- Arithmetic and Geometric means

★ Handout WS 38

nth term Arithmetic

$$a_n = a_1 + d(n-1)$$

nth term Geometric

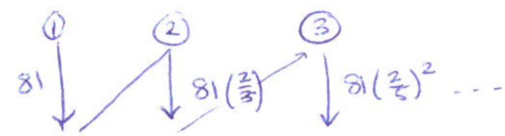
$$a_n = a_1 \cdot r^{n-1}$$

Ex 99.1 Find the 10th term in the geometric progression $x, \sqrt{2}x^2, 2x^3, \ldots$

$r = \frac{\sqrt{2}x^2}{x} = \sqrt{2}x$ so $a_n = x \cdot (\sqrt{2}x)^{n-1} \Rightarrow a_{10} = x \cdot (\sqrt{2}x)^9 = \boxed{16x^{10}\sqrt{2}}$

Ex. 99.3 A ball is dropped from a height of 81 inches. One each bounce, the ball rebounds two fifths of the distance it fell. How far does the ball fall on its 6th fall?

$$a_n = 81\left(\tfrac{2}{5}\right)^{n-1}$$



$$\boxed{a_6 = 81\left(\tfrac{2}{5}\right)^5 \approx 0.82944 \text{ inches}}$$

Arithmetic Mean of x and y:

$x, \boxed{\frac{x+y}{2}}, y$

a_1, a_1+d, a_1+2d

$d = \frac{y-x}{2}$

$a_1 + d = \frac{x+y}{2}$

Geometric mean of x and y:

$x, \boxed{\pm\sqrt{xy}}, y$

a_1, a_1r, a_1r^2

$r = \pm\sqrt{\frac{y}{x}}$

$a_1 r = x\left(\pm\sqrt{\frac{y}{x}}\right) = \pm\sqrt{xy}$

Ex 99.6 The positive geometric mean of two numbers is 8 and the difference between them is 30. Find the numbers.

① $8 = \sqrt{x \cdot y}$

② $x - y = 30$

$y = x - 30$

$64 = x^2 - 30x$

$x^2 - 30x - 64 = 0$

$(x-32)(x+2) = 0$

$\Rightarrow$ $x = 32$ or $x = -2$; $y = -2$ or $y = 32$