Pre-Calc AB

Lesson 86

2/8/16

Agenda: 2/8/16
Lesson 86
Arithmetic Progressions
Arithmetic Means

★ Test on Wednesday
lesson 1-83

★ Handout WS 34

Terminology:

Sequence: group of numbers arranged in a definite order Ex: 3, 5, 7, 9, 11

finite - finite number of members Ex: 1, 2, 3

infinite - no end to sequence Ex: 1, 2, 3, ...

Progression: Sequence in which each term depends on the proceding term

Arithmetic Progression - add same constant term to form succeeding term
Ex: 2, 4, 6, 8, 10 Ex. 3, 0, -3, -6,

Common diffecnce - difference between terms in an arithmetic Progression
Ex: above Cd = 2 cd = -3

arithmetic means - terms inbetween the end terms. Ex: above 4, 6, 8

Pattern:

1	2	3	4	5	n
2	4	6	8	10	$2n$
a_1	a_1+d	a_1+2d	a_1+3d	a_1+4d	$a_1+(n-1)d$

Ex. Find the 25th term in the arithmetic sequence whose first term is 12
86.2 and whose common difference is -6.

$$a_{25} = a_1 + (25-1)d = 12 + (24)(-6) = \boxed{-132}$$

Ex. Insert 3 arithmetic means between -2 and 3.

1 2 3 4 5

$a_1 = -2$ a_1+d a_1+2d a_1+3d $a_1+4d = 3$

$d = \frac{5}{4}$ so $a_2 = -2 + \frac{5}{4} = \boxed{-\frac{3}{4}}$ $a_3 = -2 + \frac{5}{2} = \boxed{\frac{1}{2}}$ $a_4 = -2 + \frac{15}{4} = \boxed{\frac{7}{4}}$

Ex. 86.4 Write the first 5 terms of an arithmetic sequence in which $a_{17} = -40$ and $a_{28} = -73$.

$a_{17} = a_1 + 16d$ and $a_{28} = a_1 + 27d$

$$\begin{aligned} a_1 + 27d &= -73 \\ -a_1 - 16d &= 40 \\ \hline 11d &= -33 \end{aligned}$$

$a_1 = 8$ ⇐ $d = -3$

$\boxed{8, 5, 2, -1, -4}$

Pre-Calc AB $\boxed{\text{Lesson 87}}$ 2/9/16

Agenda: 2/9/16
Lesson 87
Sum & difference Identities
Tangent identities

★ Test 10 tomorrow

Sum & Difference Identities:

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin(A-B) = \sin A\cos B - \sin B\cos A$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B$

Ex. Simplify $\cos(\theta + \frac{\pi}{4})$

$$\cos(\theta + \tfrac{\pi}{4}) = \cos\theta\cos\tfrac{\pi}{4} - \sin\theta\sin\tfrac{\pi}{4}$$
$$= \tfrac{\sqrt{2}}{2}\cos\theta - \tfrac{\sqrt{2}}{2}\sin\theta$$
$$= \boxed{\tfrac{\sqrt{2}}{2}(\cos\theta - \sin\theta)}$$

Ex. Find $\sin 15°$ by writing $15°$ as a difference of two known angles.

$$\sin 15° = \sin(60° - 45°)$$
$$= \sin 60°\cos 45° - \sin 45°\cos 60°$$
$$= \tfrac{\sqrt{3}}{2}\cdot\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\,\tfrac{1}{2}$$
$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

Ex. Simplify $\sin(\theta + \frac{\pi}{2})$

$$\sin(\theta + \tfrac{\pi}{2}) = \sin\theta\cos\tfrac{\pi}{2} + \sin\tfrac{\pi}{2}\cos\theta$$
$$= \sin\theta\cdot 0 + \cos\theta\cdot 1$$
$$= \boxed{\cos\theta}$$

Ex. Develop an identity for $\tan(A+B)$ then $\tan(A-B)$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B} \div \frac{\frac{1}{\cos A\cos B}}{\frac{1}{\cos A\cos B}} = \boxed{\frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}}$$

$$\tan(A-B) = \tan(A + (-B)) = \frac{\tan A + \tan(-B)}{1 - \tan A\tan(-B)} = \boxed{\frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}}$$

Pre-Calc AB — Lesson 88 — 2/11/16

Agenda: 2/11/16

Lesson 88

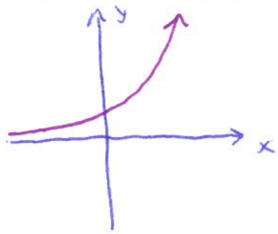
Exponential functions growth and decay

★ Handout WS 35

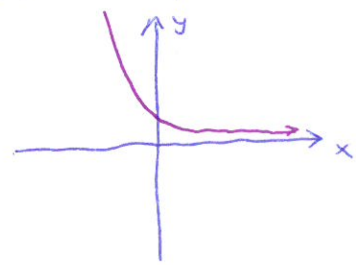
★ Test back after lesson

Exponential functions:

$f(x) = a^x$, $a > 1$



$f(x) = a^x$, $0 < a < 1$



Important Exponential Functions:

① $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ $\boxed{\text{Interest Rate formula}}$

r = Interest rate

n - # times compounded

t time

P = initial amount

② $A(t) = A_0 e^{kt}$ $\boxed{\text{Exponential growth and decay}}$ + $\boxed{\text{Compounded Continuously}}$

A_0 = initial amount

k = constant of proportionality

$k > 0 \Rightarrow$ Growth

$k < 0 \Rightarrow$ Decay

Ex. You invest \$1000 in a bank at a 5% interest rate compounded monthly for 5 years. How much will you have at the end of 5 years?

$$A(5) = 1000\left(1 + \frac{0.05}{12}\right)^{12(5)} \approx \boxed{\$1283.36}$$

Ex. 88.1 The number of bacteria present at noon was 400, and 9 hours later the bacteria numbered 800. Assume exp. growth and find the number of bacteria present at noon the next day.

$$A(t) = 400e^{kt} \Rightarrow 800 = 400e^{k\cdot 9} \Rightarrow k = \tfrac{1}{9}\ln(2) \approx 0.077$$

$$A(t) = 400e^{\frac{1}{9}\ln(2)t} \Rightarrow A(24) = 400e^{\frac{1}{9}\ln(2)(24)} \approx \boxed{2539 \text{ bacteria}}$$

Ex. 88.4 A radioactive substance decays exponentially. After 100 years, 500 grams is left and after 200 years, only 300 grams remains. Find the amount of the substance that was initially present and its half life.

$$500 = A_0e^{100k} \text{ and } 300 = A_0e^{200k} \Rightarrow \frac{5}{3} = e^{-100k} \Rightarrow \boxed{k = -\frac{1}{100}\ln\left(\frac{5}{3}\right) \approx -0.005}$$

$$A_0 = \frac{500}{e^{100k}} \approx \boxed{824.36\text{ g}} \qquad \tfrac{1}{2}A_0 = A_0e^{kt} \Rightarrow t = \frac{1}{k}\ln\left(\tfrac{1}{2}\right) \approx \boxed{138.63 \text{ years}}$$

Pre-Calc AB

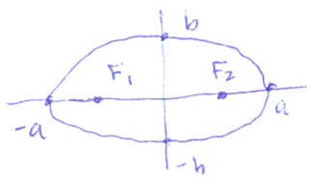
Lesson 89 + 90

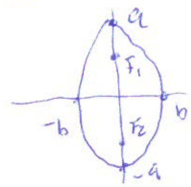
2/12/16

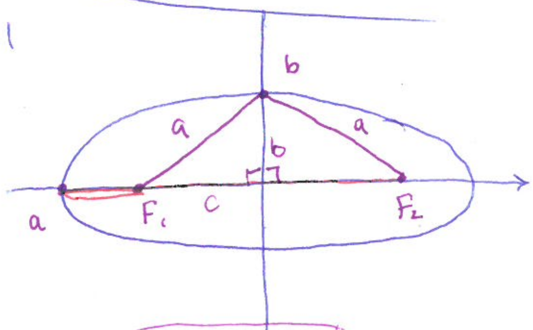
Agenda: 2/12/16
Lesson 89 + 90

The Ellipse 2 (89)
Double Angle Identities (90)
Half Angle Identities (90)

Standard Form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$



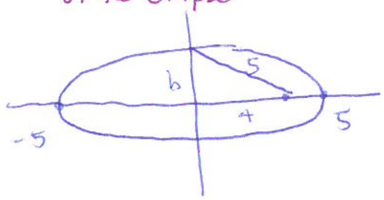




$$\boxed{a^2 = b^2 + c^2}$$

length = $2a = a + c + (a - c)$

Ex. 89.) Write the standard form of the equation of the ellipse with vertices at $(\pm 5, 0)$ and foci at $(\pm 4, 0)$.



$5^2 = 4^2 + b^2$

$\boxed{b = 3}$

$$\boxed{\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1}$$

ble Angle Identities:

$$\sin(2A) = \sin(A + A) = \sin A \cos A + \sin A \cos A = \boxed{2 \sin A \cos A}$$ ★

$$\cos(2A) = \cos(A + A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$ ★
$$= 2\cos^2 A - 1$$

$$\tan(2A) = \frac{\sin(2A)}{\cos(2A)} = \frac{2 \sin A \cos A}{\cos^2 A - \sin^2 A} \cdot \frac{1/\cos^2 A}{1/\cos^2 A} = \boxed{\frac{2 \tan A}{1 - \tan^2 A}}$$

Half-Angle Identities:

$$\sin^2 A = \frac{1 - \cos(2A)}{2} \Rightarrow \sin A = \pm\sqrt{\frac{1 - \cos(2A)}{2}} \Rightarrow \boxed{\sin\left(\tfrac{1}{2}A\right) = \pm\sqrt{\frac{1 - \cos A}{2}}}$$

$$\cos^2 A = \frac{\cos(2A) + 1}{2} \Rightarrow \cos A = \pm\sqrt{\frac{\cos(2A) + 1}{2}} \Rightarrow \boxed{\cos\left(\tfrac{1}{2}A\right) = \pm\sqrt{\frac{1 + \cos A}{2}}}$$