Pre-Calc AB

Lesson 81

1/29/16

Agenda: 1/29/16

Lesson 81

Law of Cosines

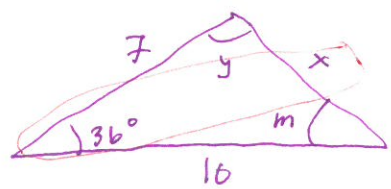
★ Handout WS 31

To Solve a Triangle need one side and two other parts.

- Law of Sines : Know angle and opposite side $\frac{A}{\sin A} = \frac{B}{\sin B} = \frac{C}{\sin C}$
- Law of Cosine: pair not Known

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

Ex. 81.1 Solve for the unknown parts.



$$x^2 = 7^2 + 10^2 - 2(7)(10)\cos 36° \quad \text{[Law of Cosine]}$$

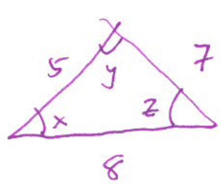
$$x^2 = 149 - 140\cos 36° \approx 35.74$$

$$\boxed{x \approx 5.98}$$

$$\frac{5.98}{\sin 36°} = \frac{7}{\sin m} \Rightarrow \boxed{m = 43.47°} \quad \boxed{y = 100.61°}$$

(or 136.53°) Not possible

Ex. 81.2 Solve



$$7^2 = 5^2 + 8^2 - 2(5)(8)\cos x$$

$$\frac{1}{2} = \frac{49 - 25 - 64}{-80} = \cos x$$

$\boxed{x = 60°}$ or 300° Not possible as $y > x$

Law of Sines

$$\boxed{\begin{aligned} y &\approx 81.77° \\ z &\approx 38.23° \end{aligned}}$$

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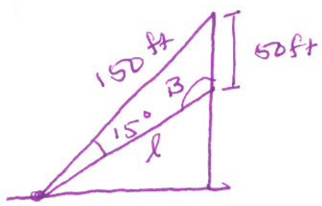
Ex. 81.4



Two cables are attached to a vertical pole on the ground as shown. Find the length of the shorter cable, ℓ.

Law of Sines: $\frac{50}{\sin 15^\circ} = \frac{150}{\sin B} = \frac{\ell}{\sin L}$

So $\sin B = 3 \sin 15^\circ \Rightarrow B \approx 50.94^\circ$ or 129.06°

Since $150 > \ell \Rightarrow B > L \Rightarrow B = 129.06^\circ$

$\Rightarrow \boxed{L = 35.94^\circ}$

So $\ell = \frac{50 \sin 35.94^\circ}{\sin 15^\circ} \approx \boxed{113.39 \text{ ft.}}$

Pre-Calc AB

Lesson 82

2/1/16

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lesson 82
Taking the log of Exponential Equations

☆ Taking the logarithm of is an operation
So taking the logarithm of an expression means the expression is in the logarithmic function.

Ex. Take the logarithm base 5 of the expression

$$x^2 - 3 = x$$

$$\log_5(x^2-3) = \log_5(x)$$

☆ The power rule of logarithms allow us to bring exponents down which is very helpful when the variable is in the exponent, such as exponential equations.

$$\log_b(a^x) = x \cdot \log_b(a)$$

Ex. 82.1 Solve: $10^{-2x+2} = 8$

Take the common log of both sides:

$$\log(10^{-2x+2}) = \log 8 \Rightarrow -2x+2 = \log 8$$

$$\boxed{x = \frac{\log 8 - 2}{-2}} \approx 0.5485$$

Ex. 82.3 Solve: $5^{2x-1} = 6^{x-2}$

Take natural log of both sides:

$$\ln(5^{2x-1}) = \ln(6^{x-2}) \Rightarrow (2x-1)\ln(5) = (x-2)\ln(6)$$

$$\boxed{x = \frac{\ln(5) - 2\ln(6)}{2\ln(5) - \ln(6)} \approx -1.383}$$

Ex. 82.4 Solve: $4 = 6e^{2x+3}$

$$\ln(4) = \ln(6)\cdot(2x+3)$$

$$\boxed{x = \frac{\ln(4) - 3\ln(6)}{2\ln(6)} \approx -1.7028}$$

Pre-Calc AB

Lesson 83

2/2/16

Agenda: 2/2/16
Lesson 83
Simple Probability
Independent Events

★ **Probability** - study of outcomes that have an equal chance of occurring.
(i.e. a fair coin should come up heads as often as it comes up tails)

Activites are called experiments: Ex. flipping coins, rolling dice, selecting cards etc.

Individual results called outcomes: Ex. one coin flip, one dice roll, one card selected

Sample space - set of outcomes of an experiment.

Events - subsets of the sample space

The probability of an event E occurring is:

$$P(E) = \frac{\text{number of outcomes that are } E}{\text{Total number of outcomes in sample space}}$$

Ex. Probability of getting a head when flipping a fair coin is

$$P(H) = \boxed{\frac{1}{2}}$$

Probability of rolling a 5 on a fair 6-sided dice:

$$P(5) = \boxed{\frac{1}{6}}$$

Probability of rolling a number greater than 2:

$$P(>2) = \frac{4}{6} = \boxed{\frac{2}{3}}$$

Ex. 83.1 Two fair dice are rolled. What is the probability of (a) a 7 and (b) >8

Total possible outcomes $= 6 \cdot 6 = 36$

(a) $P(7) = \dfrac{(6,1),(5,2),(4,3),(3,4),(2,5),(1,6)}{36} = 6/36 = \boxed{1/6}$

(b) $P(>8) = P(\text{All}) - P(<8)$
$= \dfrac{(3,6),(4,5),(5,4),(3,6)\ (4,6)\ (6,4)\ (5,6)(6,5)\ (6,6)\ (6,6)}{36}$
$= \dfrac{10}{36} = \boxed{\dfrac{5}{18}}$

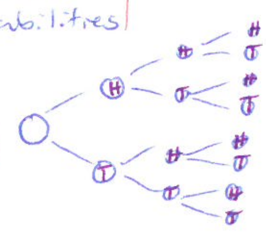
Independent Events - events that do not affect one another

Probability of independent events = Product of the individual event probabilities

Ex. A fair coin is tossed 3 times. What is the probability of getting a) all heads b) 2 tails and 1 head

(a) $P(3\text{ heads}) = \dfrac{1}{2}\dfrac{1}{2}\dfrac{1}{2} = \dfrac{1}{2\cdot2\cdot2} = \boxed{\dfrac{1}{8}}$

b) $P(2\text{ tails}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$



Pre-Calc AB $\boxed{\text{Lesson 84}}$ 2/4/16

Agenda: 2/4/16
Lesson 84
Factorable Expressions
Sketching Sinusoids

Factor:

$\tan^2\theta - 1 = (\tan\theta - 1)(\tan\theta + 1)$

$\sin x - \sin x \cos^2 x = \sin x(1-\cos x)(1+\cos x)$

$\csc^4 p - \cot^4 p = (\csc^2 p + \cot^2 p)(\csc^2 p - \cot^2 p)$

$1 - 2\sin^2 x + \sin^4 x = (1-\sin^2 x)(1-\sin^2 x)$

$\sin^3 x - \cos^3 x = (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)$

Ex. 84.2 Show $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} + \tan^2 x = \sec^2 x$

LHS $= \dfrac{\cancel{(\sec^2 x + \tan^2 x)}(\sec^2 x - \tan^2 x)}{\cancel{\sec^2 x + \tan^2 x}} + \tan^2 x$ [Diff. of squares]

$= \sec^2 x - \tan^2 x + \tan^2 x$ [Cancelation]

$= \sec^2 x$ [algebra]

= RHS ✓ ∎

Ex 84.1 Show $\sin x - \sin x \cos^2 x = \sin^3 x$

LHS $= \sin x(1-\cos^2 x)$ [factoring]

$= \sin x \sin^2 x$ [pythagorean identity]

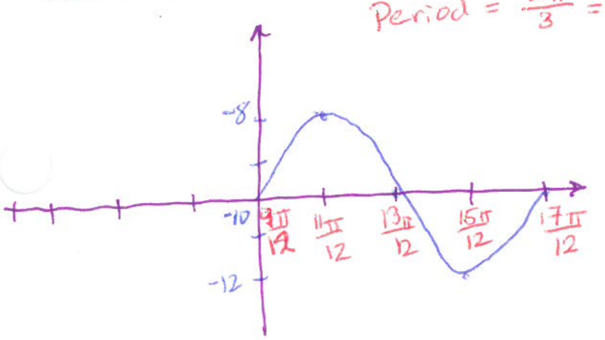
$= \sin^3 x$

= RHS ✓ ∎

Sketch:

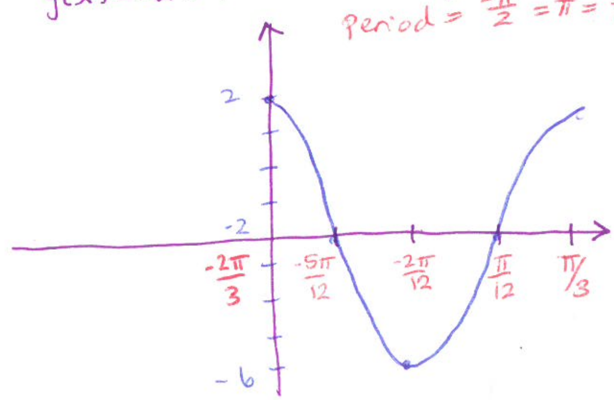
$f(x) = -10 + 2\sin\left(3x - \frac{3\pi}{4}\right) = -10 + 2\sin 3\left(x - \frac{\pi}{4}\right)$

Period $= \frac{2\pi}{3} = \frac{8\pi}{12}$



$g(x) = -2 + 4\cos 2\left(x + \frac{2\pi}{3}\right)$

Period $= \frac{2\pi}{2} = \pi = \frac{3\pi}{3} = \frac{12\pi}{12}$



Pre-Calc AB

$\boxed{\text{lesson 85}}$

2/5/16

Agenda: 2/5/16
lesson 85
Advanced Trig Equations

Ex. 85.1 Solve $3\tan^2\theta = 7\sec\theta - 5$ if $0 \le \theta < 2\pi$

$$3(\sec^2\theta - 1) = 7\sec\theta - 5 \qquad [\tan^2\theta + 1 = \sec^2\theta]$$

$$3\sec^2\theta - 7\sec\theta + 2 = 0$$

$$(3\sec\theta - 1)(\sec\theta - 2) = 0$$

$\sec\theta = \frac{1}{3}$ $\qquad$ $\sec\theta = 2$

$\cos\theta = 3$ $\qquad$ $\cos\theta = \frac{1}{2}$

No Solutions $\qquad$ $\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

Pythagoream Identities:

1. $\cos^2 x + \sin^2 x = 1$
2. $1 + \tan^2 x = \sec^2 x$
3. $\cot^2 x + 1 = \csc^2 x$

Ex. 85.2 Solve $2\sin^2 x = 3 + 3\cos x$ if $0 \le x < 2\pi$

$$2(1 - \cos^2 x) = 3 + 3\cos x$$

$$2\cos^2 x + 3\cos x + 1 = 0$$

$$(2\cos x + 1)(\cos x + 1) = 0$$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3} \qquad x = \pi}$

Ex. Solve $\cot^3(3\theta) - 3\cot(3\theta) = \csc^2(3\theta) - 4$ if $0 \le \theta < 2\pi$

$$\cot^3(3\theta) - \cot^2(3\theta) - 3\cot(3\theta) + 3 = 0$$

$$\cot^2(3\theta)[\cot(3\theta) - 1] - 3[\cot(3\theta) - 1] = 0$$

$$(\cot(3\theta) - 1)(\cot^2(3\theta) - 3) = 0$$

$\cot(3\theta) = 1$ $\qquad$ $\cot(3\theta) = \pm\sqrt{3}$

$3\theta = \frac{\pi}{4} + \pi k$ $\qquad$ $3\theta = \frac{\pi}{6} + \pi k$

$3\theta = \frac{5\pi}{6} + \pi k$

$\theta = \frac{\pi + 4\pi k}{12}$

$\theta = \frac{\pi + 6\pi k}{18}$

$\theta = \frac{5\pi + 6\pi k}{18}$

$\boxed{\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}}$

$\boxed{\theta = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}}$

$\boxed{\theta = \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}, \frac{23\pi}{18}, \frac{29\pi}{18}, \frac{35\pi}{18}}$