Pre-Calc AB

Lesson 78

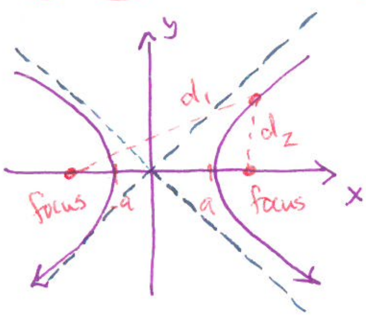
1/25/16

Agenda: 1/25/16

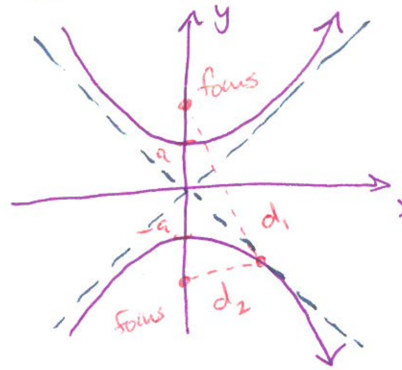
Lesson 78

The Hyperbola

★ Test 9 on Wednesday
Lessons 1-75



or



Locus Definition:

The hyperbola is the locus of all points such that the absolute value of the difference of the distance from any point on the hyperbola to two fixed points is a constant.

$$|d_1 - d_2| = \text{Constant}$$

Standard Form Centered at the Origin:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad\qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

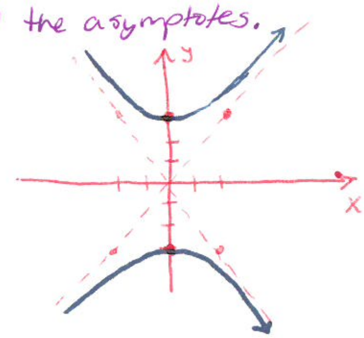
Foci on the x-axis — Foci on the y-axis

Slant Asymptotes: $\frac{x^2}{a^2} = \frac{y^2}{b^2}$ so $\boxed{y = \pm \frac{b}{a}x}$ or $\boxed{y = \pm \frac{a}{b}x}$

Ex. Sketch the hyperbola $\frac{y^2}{9} - \frac{x^2}{4} = 1$ and find the equations of the asymptotes.

$$\frac{y^2}{3^2} - \frac{x^2}{2^2} = 1$$

Asymptotes: $y = \pm\frac{3}{2}x$

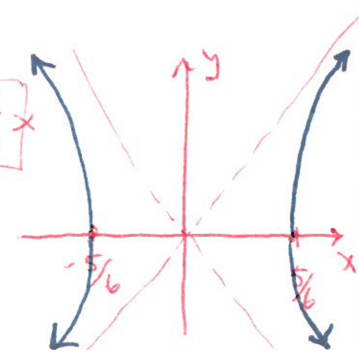


Ex. Write $25x^2 - 9y^2 - 36 = 0$ in standard form, find the vertices and the asymptotes.

$$\frac{5^2x^2}{6^2} - \frac{3^2y^2}{6^2} = 1 \Rightarrow \boxed{\frac{x^2}{(6/5)^2} - \frac{y^2}{2^2} = 1}$$

Vertices: $\boxed{(6/5, 0) \text{ and } (-6/5, 0)}$

Asymptotes: $y = \pm\frac{10}{6}x = \boxed{\pm\frac{5}{3}x}$



Pre-Calc AB

$\boxed{\text{Lesson 79}}$

1/26/16

Agenda: 1/26/16

Lesson 79

De Moivre's Theorem

Roots of Complex Numbers

★ Handout WS 30

★ Test 9 tomorrow

Evaluate: $z = (a+bi)^n$

Rewrite: $z = r\,\text{cis}\,\theta = re^{i\theta}$ so $z^n = r^n(\text{cis}\,\theta)^n = (re^{i\theta})^n$

$= r^n(e^{i\theta})^n$

$= r^n e^{in\theta}$

$= r^n\,\text{cis}(n\theta)$

De Moivre's Theorem:

$$\boxed{(r\,\text{cis}\,\theta)^n = r^n\,\text{cis}(n\theta)}$$

Example 79.2 Find $(1+i)^{13}$ $r=\sqrt{2}$ $\tan\theta = 1$ so $\theta = \frac{\pi}{4}$

$$(1+i)^{13} = \left(\sqrt{2}\,\text{cis}\,\frac{\pi}{4}\right)^{13} = (\sqrt{2})^{12}\cdot\sqrt{2}\ \text{cis}\,\frac{13\pi}{4}$$

$$= 2^6\cdot\sqrt{2}\ \text{cis}\,\frac{5\pi}{4}$$

$$= 64\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

$$= \boxed{-64-64i}$$

We say $2 = 8^{1/3}$ is the real cube root of 8 however there are 3 cube roots of 8.

$$(8)^{1/3} = (8\,\text{cis}(0+2\pi k))^{1/3} = 2^{3}\,\text{cis}\left(\frac{2\pi k}{3}\right)$$

So cube roots of 8 are: $\boxed{2^{3}\text{cis}\,0,\ 2^{3}\text{cis}\,\frac{2\pi}{3},\ 2^{3}\text{cis}\,\frac{4\pi}{3}}$

$$= \sqrt[3]{2},\ \sqrt[3]{2}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right),\ \sqrt[3]{2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$$

★ Every Complex number except zero has n, nth roots!

Ex. Find the 4th roots of $81\,\text{cis}\,\frac{\pi}{3}$. Check by multiplying.

$$\left(81\,\text{cis}\left(\frac{\pi}{3}+2\pi k\right)\right)^{1/4} = 3\,\text{cis}\left(\frac{\pi}{12}+\frac{6\pi k}{12}\right)$$

$\left(3\,\text{cis}\,\frac{\pi}{12}\right)^4 = 81\,\text{cis}\,\frac{\pi}{3}$ ✓ $\left(3\,\text{cis}\,\frac{7\pi}{12}\right)^4 = 81\,\text{cis}\,\frac{7\pi}{3} = 81\,\text{cis}\,\frac{\pi}{3}$ ✓

$\left(3\,\text{cis}\,\frac{13\pi}{12}\right)^4 = 81\,\text{cis}\,\frac{13\pi}{3} = 81\,\text{cis}\,\frac{\pi}{3}$ ✓ $\left(3\,\text{cis}\,\frac{19\pi}{12}\right)^4 = 81\,\text{cis}\,\frac{19\pi}{3} = 81\,\text{cis}\,\frac{\pi}{3}$ ✓

Ex. 79.4 Find five fifth roots of i.

$$(0+i)^{1/5} = \left(1\,\text{cis}\left(\pi/2+2\pi k\right)\right)^{1/5} = 1\,\text{cis}\left(\frac{\pi}{10}+\frac{4\pi k}{10}\right)$$

5th roots: $\boxed{1\,\text{cis}\left(\frac{\pi}{10}\right),\ 1\,\text{cis}\left(\frac{\pi}{2}\right),\ 1\,\text{cis}\left(\frac{9\pi}{10}\right),\ 1\,\text{cis}\left(\frac{13\pi}{10}\right),\ 1\,\text{cis}\left(\frac{17\pi}{10}\right)}$

Pre-Calc AB

Lesson 80

1/28/16

Agenda: 1/28/16
Lesson 80
Trigonometric Identities

★ Test back after lesson

Pythagorean Identities (Sooo Important!)

★ $\boxed{\sin^2\theta + \cos^2\theta = 1}$ $\quad \sin^2\theta + \cos^2\theta = y^2 + x^2 = 1$ [On the unit circle!]

$\boxed{1 + \cot^2\theta = \csc^2\theta}$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$ $\quad \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

Ex. 80.1 Show $\dfrac{\sec^2 x - \tan^2 x}{1 + \cot^2 x} = \sin^2 x$

$$LHS = \frac{\sec^2 x - \tan^2 x}{1 + \cot^2 x} = \frac{1}{\csc^2 x}$$ [$\sec^2 x - \tan^2 x = 1$, $1 + \cot^2 x = \csc^2 x$ Pythagorean Identities]

$$= \sin^2 x$$ [def of csc]

$$= RHS \checkmark \blacksquare$$

Ex. 80.2 Show $\dfrac{1}{\tan A} + \tan A = \sec A \csc A$

$$LHS = \frac{1}{\tan A} + \tan A = \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$$ [def of tan]

$$= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$$ [Common denominator]

$$= \frac{1}{\sin A \cos A}$$ [Pythagorean Identity]

$$= \csc A \sec A = RHS \checkmark$$ [def of csc and sec] $\blacksquare$

Ex. 80.4 Show $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2\sec A$

$$LHS = \frac{\cos A}{1 + \sin A} \cdot \frac{\cos A}{\cos A} + \frac{1 + \sin A}{\cos A} \cdot \frac{(\sin A + 1)}{(\sin A + 1)}$$ [multiply by fancy 1 / Common denom]

$$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1 + \sin A)} = \frac{2(1 + \sin A)}{\cos A(1 + \sin A)} = 2\sec A \blacksquare$$