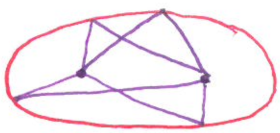
Pre-Calc AB Lesson 71 12/14/15

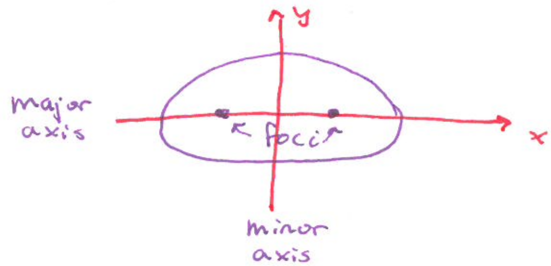
Agenda: 12/14/15

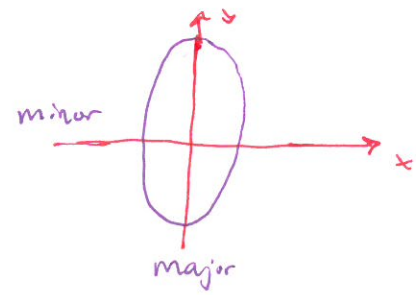
The Ellipse

★ Handout Pre-Comp Study Guide

Definition - the ellipse is the locus of points such that the sum of the distances from a point on the ellipse to two foci is constant.

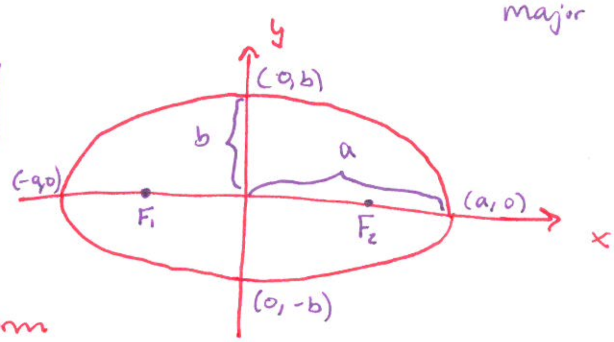






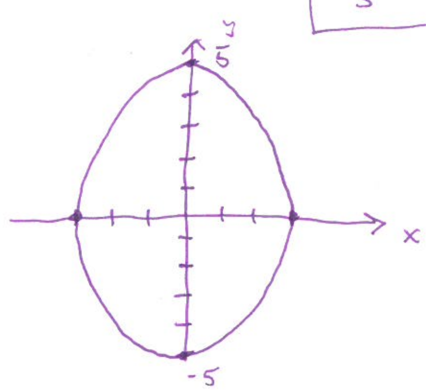
Ellipse with center at the origin:

Standard form: $\boxed{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$



Ex. Write the equation in standard form and graph the ellipse: $5x^2 + 3y^2 = 15$

$$\frac{5x^2}{15} + \frac{3y^2}{15} = 1 \qquad \boxed{\frac{x^2}{3} + \frac{y^2}{5} = 1}$$



Pre-Calc AB

Lesson 72

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Lesson 72

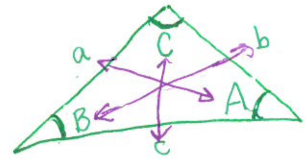
One side + 2 parts

Law of Sines

★ Handout WS 26

★ HW For Friday Bring a White Elephant Gift.

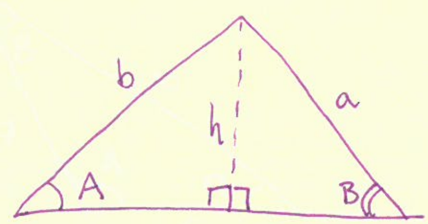
★ If the length of one side and the measures of two other parts of a triangle are known then its possible to solve for the missing parts.



★ Can't solve for missing parts if we don't know the length of at least one side.

Law of Sines: To solve a triangle with 3 known parts, two of which are a pair

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$



$h = b\sin A$

- $\{a > b\}$ only 1 angle
  $\{a > h\}$ only 1 angle
- $b > a > h$ only 2 angles
- $a < h$ No triangle

Ex Solve this triangle for the unknown parts.

$$\frac{10}{\sin A} = \frac{7}{\sin \pi/6} = \frac{x}{\sin B}$$

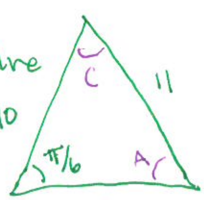
(1) $\sin A = \frac{10}{7}\sin\frac{\pi}{6} = \frac{5}{7}$

$A = \sin^{-1}($

$B = \pi - \sin^{-1}\left(\frac{5}{7}\right) - \frac{\pi}{6}$
$\approx 104.42°$
$x \approx 13.559$

$B = \sin^{-1}\left(\frac{5}{7}\right) - \frac{\pi}{6}$
$\approx 15.58°$
$x \approx 3.76$

Ex. Solve



$$\frac{\sin C}{c} = \frac{\sin A}{10} = \frac{1/2}{11}$$

$\sin A = \frac{5}{11}$

$\sin^{-1}\left(\frac{5}{11}\right) = A \approx 27.04°$

$C = \pi - \sin^{-1}\left(\frac{5}{11}\right) - \frac{\pi}{6} \approx 122.96°$

$c = 18.459$

Problem:
$A = \pi - \sin^{-1}\left(\frac{5}{7}\right) \approx 152.96$
$C = 0°$ ← BAD!!!

Ex. Solve $A = \pi/6$, $a = 4$, $b = 10$,

$$\frac{\sin B}{10} = \frac{\sin \pi/6}{4} \Rightarrow \sin B = \frac{5}{4} > 1 \quad \boxed{\text{No Solution!}}$$

Pre-Calc AB

Lesson 73 + 74

12/16/15

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Lesson 73 + 74

Regular Polygons (73)

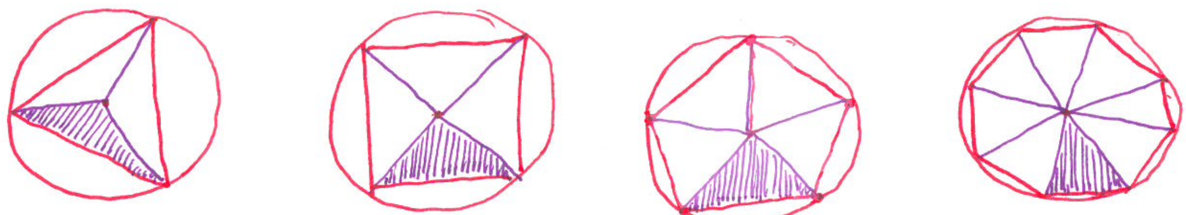
Cramer's Rule (74)

★ P2: QT Trip tomorrow after Quick Check of HW

★ P8: Christmas Party after Quick Check of HW

★ A regular polygon, of any number of sides, can be inscribed in a circle.

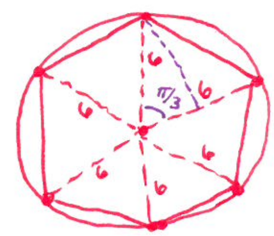
"Equalateral" - all angles equal in measure -



3 sides 4 sides 5 sides 8 sides

Area = # sides × Area of one triangle

Ex. Find the area and perimeter of a regular Hexagon inscribed in a circle of diameter 12.



Area of triangle $= \frac{1}{2}(6)\,6 \sin\frac{\pi}{3} = 9\sqrt{3}$ units

Area of Hexagon $= 6 \cdot 9\sqrt{3} = \boxed{54\sqrt{3} \text{ units}^2}$

Perimeter $= 6 \times 6 = \boxed{36 \text{ units}}$

Solve: $ax + by = c$
$dx + ey = f$

Matrix Form: $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$

$aex - bdx = ec - bf$

$x = \dfrac{ec - bf}{ae - bd}$

$bdy - aey = cd - af$

$y = \dfrac{cd - af}{bd - ae} = \dfrac{af - cd}{ae - bd}$

$$\boxed{x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}} \qquad \boxed{y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}}$$

Cramer's Rule!

Pre-Calc AB

Lesson 75

1/19/16

Agenda: 1/19/16

Lesson 75
Combinations

★ Quiz 9 tomorrow lessons 65-74

Permutations - number of ways to order a set

Ex. How many ways can 4 items be selected and arranged/ordered from 10 items?

$$P(10,4) = \frac{10!}{(10-4)!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

Combinations - number of ways to select items in which order is not considered.

Since there are $4 \cdot 3 \cdot 2 \cdot 1$ ways to arrange 4 items
then the number of unordered sets of 10 things taken 4 at a time

$$\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{5040}{24} = 210$$

$$C(10,4) = \frac{10!}{(10-4)!\,4!} = \frac{P(10,4)}{4!}$$

General Expression for the combination of n things taken r at a time:

$$\boxed{C(n,r) = {}_nC_r = \frac{n!}{(n-r)!\,r!} = \frac{P(n,r)}{r!}} = \boxed{\binom{n}{r}}$$

"read n choose r"
Most comon!

Ex. In how many ways can a committee of 7 be selected from a group of 15 students?

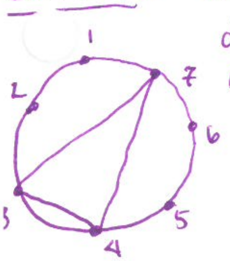
1) Does order matter? No
2) Are repetitions allowed? No

$$\Rightarrow \binom{15}{7} = \frac{15!}{7!\,8!} = \frac{15 \cdot \cancel{14} \cdot 13 \cdot \cancel{12} \cdot 11 \cdot \cancel{10} \cdot \cancel{9}^{3}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}$$

$$= 15 \cdot 13 \cdot 11 \cdot 3$$

$$= \boxed{6435}$$

Ex. 75.2 There are 7 points located on a circle, How many different triangles can be drawn using these points as vertices?



1) Does order matter? No
2) repetitions? No

select 3 points from 7

$$\binom{7}{3} = \frac{7!}{4!\,3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 7 \cdot 5 = \boxed{35}$$

Pre-Calc AB | Lesson 76 | 1/21/16

Agenda: 1/21/16

lesson 76

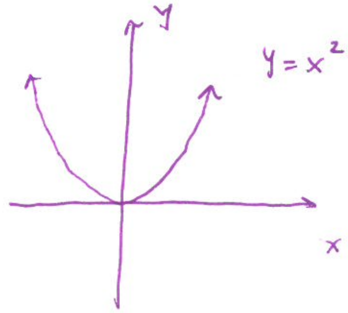
- even, odd functions
- Trigonometric identities
  - Functions of other angles
  - rules of the game

★ Handout WS 28

- A function f is an even function if $f(-x) = f(x)$
- A function f is an odd function if $f(-x) = -f(x)$

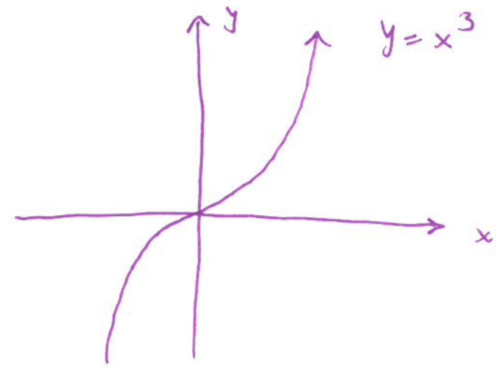
] if not then neither

Ex.



Even - Symmetric about y-axis

Ex.



Odd - Symmetric about the origin

Trig functions:

Even:

$\cos(-\theta) = \cos(\theta)$

$\sec(-\theta) = \sec(\theta)$

odd:

$\sin(-\theta) = -\sin(\theta)$ $\quad$ $\tan(-\theta) = -\tan(\theta)$

$\csc(-\theta) = -\csc(\theta)$ $\quad$ $\cot(-\theta) = -\tan(\theta)$

Ex. Is $f(x) = x^3 - 3x$ even, odd or neither?

$$f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = -(x^3 - 3x) = -f(x) \quad \boxed{\text{odd}}$$

- $\pi/2 - \theta$ or $90° - \theta$ is the other angle

$\sin(\theta) = \cos(90° - \theta)$ $\quad$ $\tan(\theta) = \cot(90° - \theta)$ $\quad$ $\sec(\theta) = \csc(90° - \theta)$

$\cos(\theta) = \sin(90° - \theta)$ $\quad$ $\cot(\theta) = \tan(90° - \theta)$ $\quad$ $\csc(\theta) = \sec(90° - \theta)$

Equation vs. Identity:

Equation:

$x + 4 = 7$

"Only True for some x"

Identity:

$2x - 3 = 3x - x - 3$

"True for all x"

Simplify L→R or R→L

$3x - x - 3$

$= 2x - 3$ ✓

★ We will look at showing Trig identities

Pre-Calc AB

Lesson 76

1/21/16

Rules of the Game:

★ Do NOT Assume it is true! Show it!

- Pick the more complicated side
- Substitute an equivalent expression for any part
- Multiply by a fancy 1 = $\frac{\text{expression}}{\text{expression}}$
- Add a fancy 0 = expression − expression

★ For Trig also recomend converting to Sine and Cosine if you get stuck!

Ex. 76.2 Show $\sin x \cot x = \frac{1}{\sec x}$

$$\begin{aligned} LHS &= \sin x \cot x \\ &= \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}} && [\text{Def of Cot}] \\ &= \cos x && [\text{canceling}] \\ &= \frac{1}{\sec x} = RHS \checkmark && [\text{def of sec}] \quad \blacksquare \end{aligned}$$

Ex. Show: $\frac{\cos(-\theta)\sin(90^\circ-\theta)}{\sin(-\theta)\cos(90^\circ-\theta)} = -\cot^2\theta = -\frac{\cos^2\theta}{\sin^2\theta}$

$$\begin{aligned} LHS &= \frac{\cos(-\theta)\sin(90^\circ-\theta)}{\sin(-\theta)\cos(90^\circ-\theta)} \\ &= \frac{\cos(\theta)\sin(90^\circ-\theta)}{-\sin(\theta)\cos(90^\circ-\theta)} && [\text{Cosine is even, Sine is odd}] \\ &= \frac{\cos\theta\cos\theta}{-\sin\theta\sin\theta} && [\text{Other angles}] \\ &= -\frac{\cos^2\theta}{\sin^2\theta} && [\text{Combining}] \\ &= -\cot^2\theta = RHS \checkmark && [\text{definition of Cot}] \quad \blacksquare \end{aligned}$$

Pre-Calc AB | Lesson 77 | 1/22/16

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Lesson 77
Binomial Expansions

★ Handout WS 29

• Binomial – Expression with two terms Ex. $a+b$, xy^2+3z^3

★ Often have to expand binomials to powers... want a fast way to do it

Pascal's Triangle

$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Example: Use Pascal's Triangle to write the expansion for $(x+y)^8$

$(a+b)^6$: 1 6 15 20 15 6 1
$(a+b)^7$: 1 7 21 35 35 21 7 1
$(a+b)^8$: 1 8 28 56 70 56 28 8 1

$\boxed{a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8}$

Example: Find the 6th term of $(2x+z^2)^7$

$$21(2x)^2(z^2)^5 = \boxed{84x^2z^{10}}$$

Example: Find the coefficient of the 4th term in $(3ab - \frac{1}{2}c)^6$

$$20(3ab)^3\left(-\tfrac{1}{2}c\right)^3 = 20 \cdot 27a^3b^3\left(-\tfrac{1}{8}\right)c^3$$
$$= -\frac{135}{2}a^3b^3c^3$$

$$\boxed{-\frac{135}{2}}$$