Pre-Calc AB

Lesson 6

8/7/15

Agenda: 8/7/15

HW Corrections : Lesson 5
HW leader : Me

- Lesson 6
  - Fractional Equations
  - Radical Equations
  - System of 3 equations
- Quiz #1 next Wednesday

★ You can skip construction problems. Still need to do the concept review problems

T/F There is a unique solution to a system of 2 linear equations.

Fractional Equations

★ Make it eassier, by eliminating denominators and decimals

Ex. 6.2 Solve:

$$\begin{cases} \frac{3}{7}x + \frac{2}{5}y = 11 & ① \\ 0.03x - 0.2y = -0.37 & ② \end{cases}$$

For ① eliminate denominators by multiplying both sides of the equation by the LCM of the denoms.

$LCM(7,5) = 35$ ① × 35 : $15x + 14y = 385$

For ② eliminate decimals by multiplying by 100.

② × 100 : $3x - 20y = -37$

② × −5 : $-15x + 100y = 185$ ②

$15x + 14y = 385$ ①

① + ② : $114y = 570$

$\boxed{y = 5}$

Sub y into ① $15x + 14(5) = 385$

$15x = 315$

$\boxed{x = 21}$

Check:

$\frac{3}{7}(21) + \frac{2}{5}(5)$

$= 9 + 2$

$= 11$ ✓

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Radical Equations:

1. Isolate the radical(s)
2. Square both sides
3. Repeat 1-2 if you still have a radical
4. Check your answer(s)

Ex. 6.4 Solve: $\sqrt{s-48} + \sqrt{s} = 8$

$$\left(\sqrt{s-48}\right)^2 = \left(8-\sqrt{s}\right)^2$$

$$\cancel{s} - 48 = 64 - 16\sqrt{s} + \cancel{s}$$

$$16\sqrt{s} = 112$$

$$\sqrt{s} = 7$$

$$s = 49$$

Check:

$\sqrt{49-48} + \sqrt{49}$

$= 1 + 7$

$= 8$ ✓

System of 3 Linear Equations:

- use substitution or elimination

Ex. Solve
$$\begin{cases} 2x + 2y - z = 14 & (1) \\ 3x + 3y + z = 16 & (2) \\ x - 2y = 10 & (3) \end{cases}$$

- first try to solve for one if you can
- then go to two equations two unknowns

(1) × 3: $6x + 6y - 3z = 42$

(2) × (−2): $-6x - 6y - 2z = -32$

$0 + 0 - 5z = 10$

$\boxed{z = -2}$

(1): $2x + 2y = 12$ → (1)/2: $x + y = 6$

(2): $3x + 3y = 18$

+ −(3): $-x + 2y = -10$

$3y = -4$

$\boxed{y = -\frac{4}{3}}$

$\boxed{x = \frac{22}{3}}$

Check:

$3\left(\frac{22}{3}\right) + 3\left(-\frac{4}{3}\right) + (-2)$

$= 22 - 4 - 2$

$= 16$ ✓

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Lesson 7

8/10/15

Agenda: 8/10/15

- HW Leader: Period 2: Tyler McMurrich — Period 8: Christian Bailey
- Lesson 7
  - Reasoning
  - logic
  - Contrapositive
  - Converse, Inverse
- Work on PS 7

★ Quiz 1 on Wednesday

T/F When solving systems of equations, elimination is always best.

Reasoning and Logic

Inductive Reasoning - the process of trying to find a rule.
Gather data, make observations, and then guess

Deductive Reasoning - the process of using a formulated rule.

Example: We might induce a rule that says when we flip a coin 100 times we will always have more heads than tails.

Syllogism - formal reasoning process to reach a conclusion when using two premises.

Premise: Statement that is either true or false.

- Universal Affirmative (Major Premise) - premises that affirm all members of a set posses a certain property.
- Minor Premise - identifies one member of the set.

Ex.

1. All BASIS students will take Calculus. (Major Premise)
2. John is a BASIS student. (Minor Premise)

Therefore, John will take Calculus. (Conclusion)

All 3 steps called an Argrement.

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* We will use this reasoning for geometric arguments

A valid argument gives a valid conclusion, however that does not mean that the conclusion is true.

Ex. 7.1 Is the following a valid argument?

1. All normal dogs have four legs (Major Premise)
2. That dog has four legs (Minor Premise)

Therefore, that dog is a normal dog. (Conclusion)

No, this is invalid. (2) needs to be a member of normal dogs.

Ex. 7.2 Is the following a valid argument?

1. All chickens have 3 legs. (Major Premise)
2. Penny is a chicken. (Minor Premise)

Therefore, Penny has 3 legs (Conclusion)

Yes, this is valid. However, (1) is false therefore the conclusion which is valid may be false.

Contrapositive, Converse, Inverse

If-then statements, also called implications, can be writen as:

If the hypothesis, then the conclusion.

Both the hypothesis and the conclusion are premises.

hypothesis $\longrightarrow$ conclusion

Ex. All rabbits are fast. $\underbrace{\text{If an animal is a rabbit}}_{\text{hypothesis}}^{P}$, then $\underbrace{\text{the animal is fast}}_{\text{conclusion}}^{Q}$.

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Negation of a premise: use not or non-

Ex. The animal is a rabbit.

negation: The animal is not a rabbit.

P: The animal is a rabbit

Q: The animal is fast

Implication: $P \rightarrow Q$

If the animal is a rabbit, then it is fast.

Symbol for negation

Contrapositive: $\neg Q \rightarrow \neg P$

If the animal is not fast, then it is not a rabbit.

} Both have the same truth or falsity

Converse: $Q \rightarrow P$

If the animal is fast, then its a rabbit.

Inverse: $\neg P \rightarrow \neg Q$

If the animal is not a rabbit then it is not fast.

} Both have the same truth or falsity

Bi-implication: (if and only if) $P \longleftrightarrow Q$ same as $P \rightarrow Q$ and $Q \rightarrow P$

The animal is a rabbit if and only if it is fast.

Ex. 1. The sum of all exterior angles of a convex polygon is 360°.

2. The sum of figure ABC's exterior angles is not 360°.

Therefore, figure ABC is not a convex polygon.

$(\neg Q \rightarrow \neg P)$

$P \rightarrow Q$

$\neg Q$

$\therefore \neg P$

P: The figure is a convex polygon

Q: The sum of all exterior angles is 360°

$\boxed{\text{Valid by Contrapositive of 1}}$

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Lesson 8

8/11/15

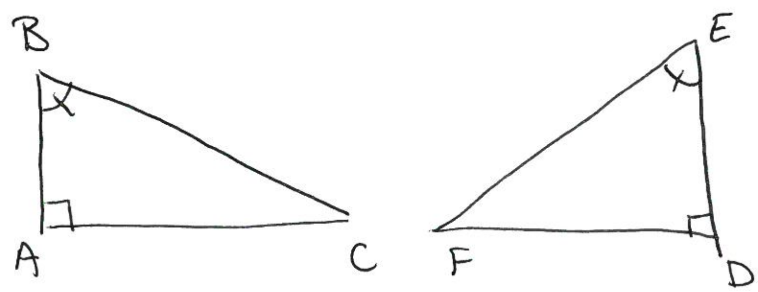
Agenda: 8/11/15

Period 2	Period 8
Tiffany Lee	Nandini Sodhi

- HW Leader:
- Lesson 8
  - Statement of Similarity
  - Proportional segments
  - Angle Bisectors
- Work on PS 8

★ Quiz 1 Tomorrow

T/F An implication and it's contrapositive have the same truth or falsity.



- Use tick marks to indicate angles having equal measure.

- $\triangle ABC \sim \triangle DEF$ "triangles are similar"

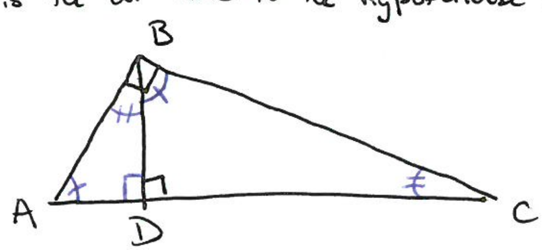
Incorrect: $\triangle ABC \sim \triangle DFE$

- $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

- ratio of corresponding sides of similar triangles are equal

Ex. 8.1 In $\triangle ABC$, segment BD is the altitude to the hypotenuse AC.

Show $\dfrac{AD}{AB} = \dfrac{AB}{AC}$



$\triangle ADB \sim \triangle ABC$

So $\dfrac{AD}{AB} = \dfrac{AB}{AC}$

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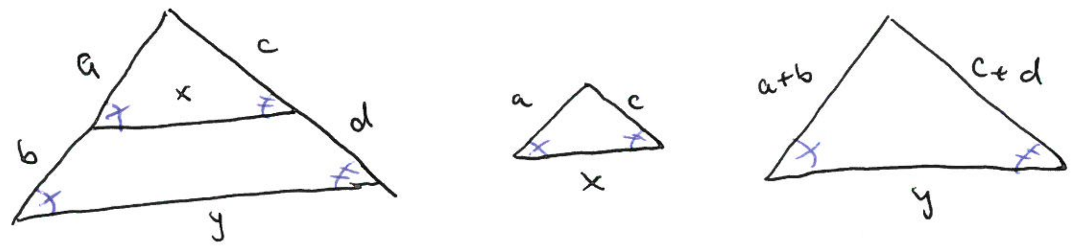
Proportional Segments

Two parallel lines cut proportional segments



Optional proof

Since the two triangles are similar we have:

$$\frac{a+b}{a} = \frac{c+d}{c} = \frac{x}{y}$$

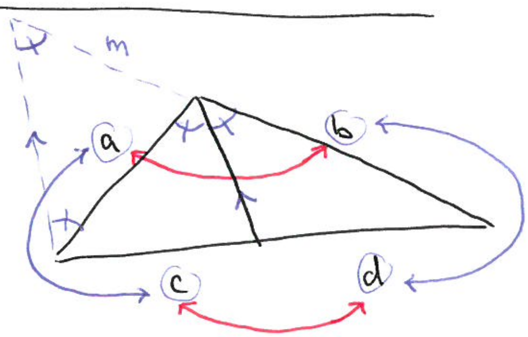
Want to show: $\frac{b}{a} = \frac{d}{c}$

$$\frac{a+b}{a} = 1 + \frac{b}{a}$$

$$\frac{c+d}{c} = 1 + \frac{d}{c}$$

so $\boxed{\frac{b}{a} = \frac{d}{c}}$

Angle Bisectors and Side Ratios



$\boxed{\frac{a}{c} = \frac{b}{d}}$

$\boxed{\frac{a}{b} = \frac{c}{d}}$

Optional proof

$$\frac{m}{c} = \frac{b}{d}$$

isosceles triangle $\Rightarrow m = a$ ✓

or

$$\frac{m}{b} = \frac{c}{d}$$

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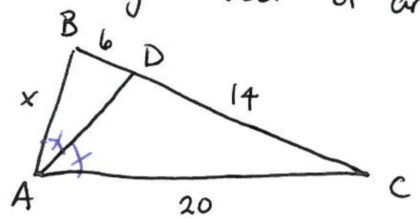
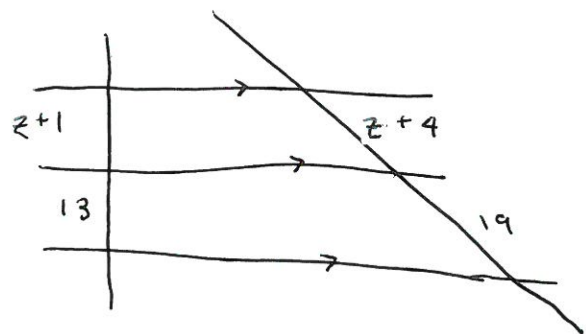
Ex. 8.4 In $\triangle ABC$, segment AD is the angle bisector of angle A. Find x:



$$\frac{x}{6} = \frac{20}{14}$$

so $\boxed{x = \frac{60}{7}}$

Ex. 8.2 Find z.



$$\frac{z+1}{13} = \frac{z+4}{19}$$

$$19z + 19 = 13z + 52$$

$$6z = 33$$

$$\boxed{z = \frac{11}{2}}$$

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Lesson 9

8/13/15

Agenda: 8/13/15

- HW Leader: Me Quiz Questions
- Lesson 9
  - Congruent figures
  - Proof outlines
- Work on PS 9

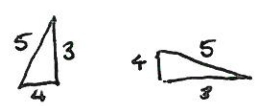
T/F Similar triangles have the same 3 interior angles.

Congruent figures

We say two figures are congruent if mentally we can place one precisely ontop of the other by flipping or rotating so that they match exactly. (page 67)

- Congruent triangles are similar triangles with scale factor 1.

Side - Side - Side (SSS)



All triangles having the same 3 side lengths are congruent.

Side - Angle - Side (SAS)

If two sides and the included angle in one triangle have the same measure as two sides and the included angle of another triangle then they are congruent

Two Angles and a Side (ASA or AAS)

☆ If we know two angles then we know all three ⇒ similar

☆ We need only one side to know the scale factor is 1 ⇒ congruent.

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<u>hypothenuse-Leg (HL)</u> ≡ (SSS) by pythagorean's identity

☆ Only for right triangles!!!

<u>Proof Outlines</u>

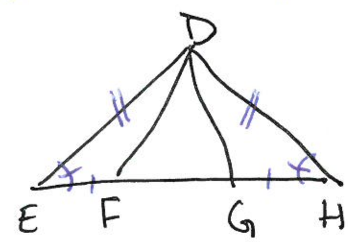
☆ Be careful to list vertices in correct order

<u>Ex. 9.2</u>

Given $\angle E \cong \angle H$ (Angles are congruent)

$\overline{EF} \cong \overline{HG}$ (segments are congruent)

Proof $\overline{DF} \cong \overline{DG}$



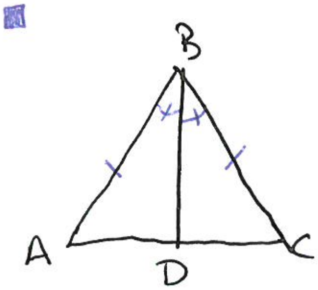
1. $\angle E \cong \angle H$ [Given]
2. $\overline{EF} \cong \overline{HG}$ [Given]
3. $\triangle EDH$ is isosceles [by 1]
4. $\overline{ED} \cong \overline{HD}$ [by 3]
5. $\triangle DEF \cong \triangle DHG$ [by SAS]
6. $\overline{DF} \cong \overline{DG}$ [by 5] ∎

<u>You try: PS 9 # 10</u>

Given $\overline{AB} \cong \overline{CB}$

$\angle ABD \cong \angle CBD$

Prove $\triangle ABD \cong \triangle CBD$



1. $\overline{AB} \cong \overline{CB}$ [Given]
2. $\angle ABD \cong \angle CBD$ [Given]
3. $\triangle ABD \cong \triangle CBD$ [SAS, 1, 2, $\overline{BD}$] ∎

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Lesson 10

8/14/15

Agenda: 8/14/15

- HW leader:
- Lesson 10
  - Equation of a line
  - Rational denoms
  - Completing the square
- Work on PS 10

Period 2: Aiko Robies

Period 8: Alistair McCallum

T/F Similar triangles are congruent.

Equations of lines:

Slope: $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$ or $\frac{y_1 - y_2}{x_1 - x_2}$

horizontal lines: slope is zero

vertical lines: slope is undefined.

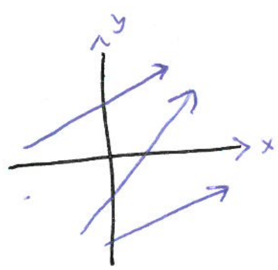
* Otherwise we will have a slope and y-intercept

where it crosses the y-axis.

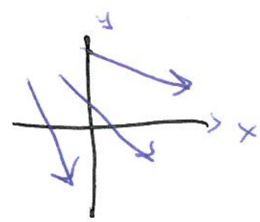
Slope-intercept form: $\boxed{y = mx + b}$

slope

y-intercept



Positive slope



Negative Slope

* Parallel lines have the same slope.

* Perpendicular lines have slope the negative reciprocal of each other.

Ex. 10.2 Find the equation (in slope-intercept form) of the line that passes through the point $(-2,5)$ and is perpendicular to $\underbrace{3y+4x-2=0}_{\text{standard form of a line}}$

Need:

1. Slope
2. y-intercept

1. Slope: perp to $y = \frac{2-4x}{3}$

So $\boxed{\text{slope} = \frac{3}{4}}$

2. $y = \frac{3}{4}x + b$ find b using $(-2,5)$

$5 = \frac{3}{4}(-2) + b$

$b = \frac{10}{2} + \frac{3}{2} = \boxed{\frac{13}{2}}$

$$\boxed{y = \frac{3}{4}x + \frac{13}{2}}$$

Rational Denominators

Real Numbers:

1. Rational numbers $\left(\frac{1}{2}, 5, \frac{17}{3}, \ldots\right)$
2. Irrational numbers $\left(\pi, e, \sqrt{142}, \sqrt[5]{17}, \ldots\right)$

Ex. 10.4 Simplify: $\frac{4+\sqrt{3}}{2-3\sqrt{3}} \cdot \frac{2+3\sqrt{3}}{2+3\sqrt{3}}$

Multiply top and bottom by the conjugate of the denom.

$$= \frac{8+2\sqrt{3}+12\sqrt{3}+9}{4-27}$$

$$= \boxed{-\frac{17+14\sqrt{3}}{23}}$$

Ex. 10.5 Simplify: $\frac{2-i^3+2i^5}{-2i+4} \cdot \frac{4+2i}{4+2i}$

Complex Conjugate

$$= \frac{2+3i}{4-2i} \cdot \frac{4+2i}{4+2i}$$

$$= \frac{8+12i+4i-6}{16+4}$$

$$= \frac{2+16i}{20} = \boxed{\frac{1}{10}+\frac{4}{5}i}$$

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Lesson 10

8/14/15

Completing the Square

- How we find roots of a quadratic if we can't factor
- How we get the quadratic formula

Ex. Solve: $\left(x-\frac{1}{2}\right)^2-3=0$

$$\left(x-\frac{1}{2}\right)^2=3$$

$$\left(x-\frac{1}{2}\right)=\pm\sqrt{3}$$

$$\boxed{x=\frac{1}{2}\pm\sqrt{3}}$$

2 Don't forget ± when taking square roots

Ex. Solve $x-2x^2=-7$

1. Write in Standard form $\qquad -2x^2+x+7=0$
2. Divide all terms by leading coefficient $\qquad x^2-\frac{1}{2}x-\frac{7}{2}=0$
3. Move constant term $\qquad \left(x^2-\frac{1}{2}x\quad\right)=\frac{7}{2}$
4. Add $\left[\frac{1}{2}(\text{coef of } x)\right]^2$ to both sides $\qquad \left(x^2-\frac{1}{2}x+\frac{1}{16}\right)=\frac{7}{2}+\frac{1}{16}$ $\qquad \left(\underbrace{\frac{1}{2}\cdot\left(-\frac{1}{2}\right)}\right)^2=\frac{1}{16}$
5. Factor the left side $\qquad \left(x-\frac{1}{4}\right)^2=\frac{57}{16}$
6. Solve $\qquad \boxed{x=\frac{1}{4}\pm\frac{\sqrt{57}}{4}}$

this will be the factor on the left

Must be a perfect square