Pre-Calc AB

# Lesson 31

9/24/15

**Agenda:** 9/24/15

HW leader: None

Lesson 31

- Symmetry
- More transformations

Warm-up: Graph by hand $f(x) = x^2 - 4x + 3$

Hint: Complete the square

Quiz back after lesson

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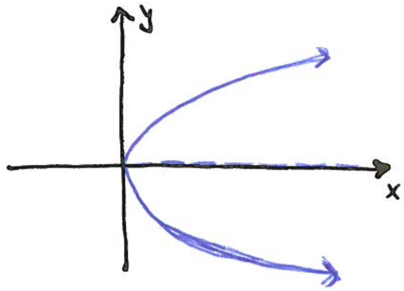
## Symmetry:

### Symmetric about x-axis

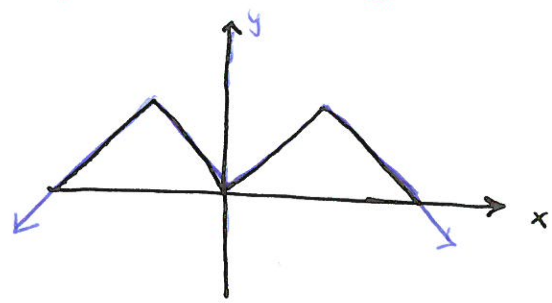
[Not a function ever]




- replacing y with $-y$ in equation stays the same

### Symmetric about y-axis

[function called even]



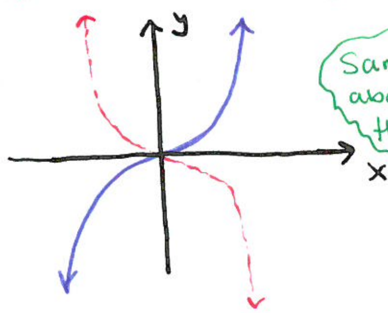

- replacing x with $-x$ in equation stays the same

**A function $f(x)$ is even if $f(-x) = f(x)$.**

### Symmetric about origin

[Graph rotated 180° is the same]

[function called odd]

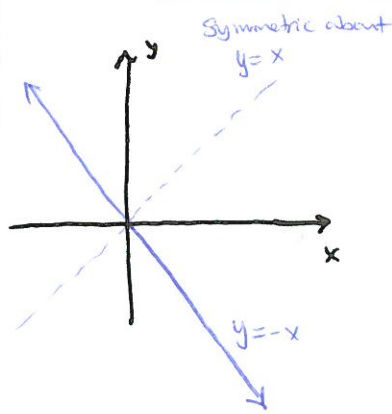


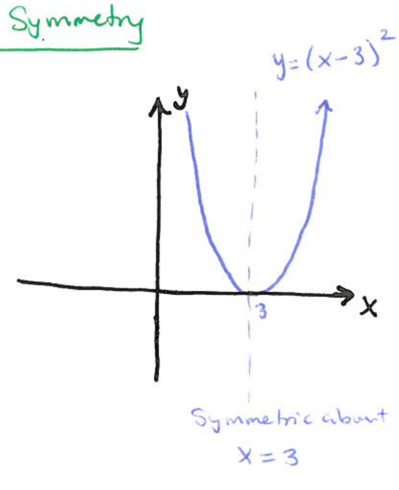

Same as reflect about y-axis then about x-axis

- replacing x with $-x$ and y with $-y$ in equation stays the same

**A function $f(x)$ is odd if $f(-x) = -f(x)$**

### Other lines of Symmetry



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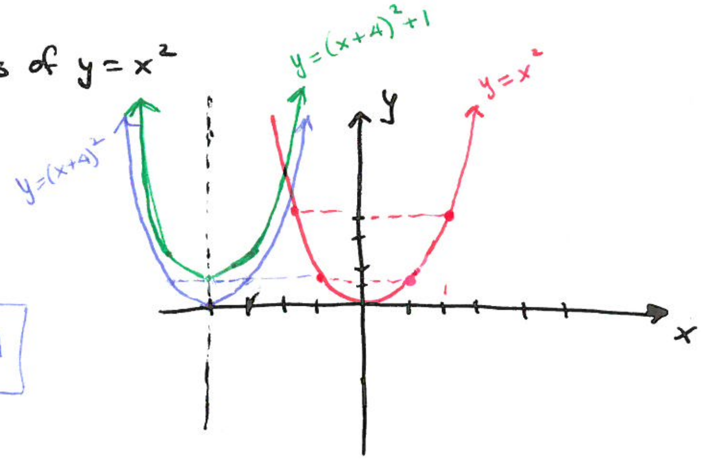
Ex. Find the line of symmetry for $f(x) = x^2 + 8x + 17$ by

1. Completing the square
2. Graphing using transformations of $y = x^2$

1. $f(x) = (x+4)^2 + 1$

left 4    up 1

2.




$\boxed{\text{The line of symmetry is } x = -4}$

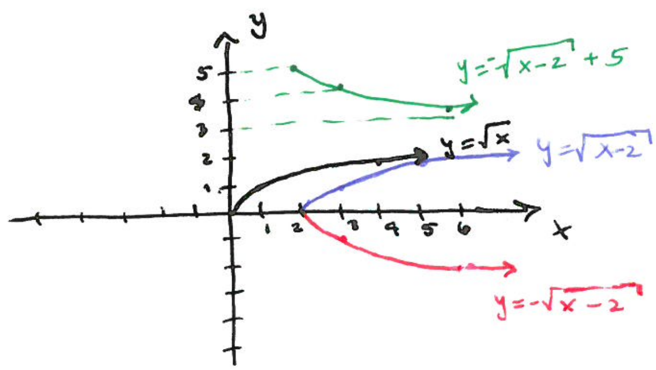
Ex. Find the domain and range of $g(x) = -\sqrt{x-2} + 5$

$y = \sqrt{x}$    D: $[0, \infty)$    R: $[0, \infty)$

(Left 2)   $y = \sqrt{x-2}$    D: $[2, \infty)$    R: $[0, \infty)$

(reflect y-axis)   $y = -\sqrt{x-2}$    D: $[2, \infty)$    R: $(-\infty, 0]$

(up 5)   $y = -\sqrt{x-2}$    $\boxed{D: [2, \infty) \quad R: (-\infty, 5]}$



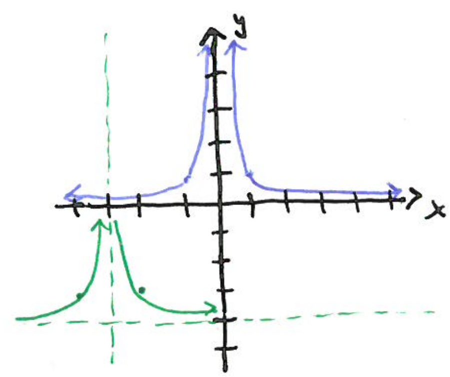

Your Turn

Find the domain, range and symmetry for

$$h(x) = \frac{1}{(x+3)^2} - 4$$

D: $(-\infty, -3) \cup (-3, \infty)$    R: $(4, \infty)$

Line of Symmetry: $x = -3$



Pre-Calc AB

**Lesson 32**

9/25/15

Agenda: 9/25/15

| | Period 2 | Period 8 |
|---|---|---|
| HW leader: | Sabrina | Nandini |

Lesson 32 part 1

Inverse Functions

- Handout WS 8

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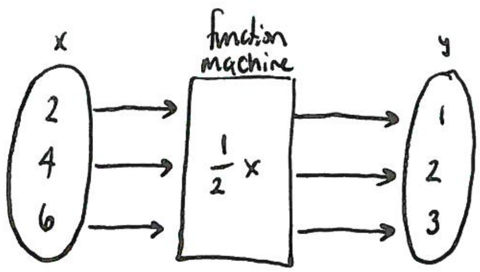
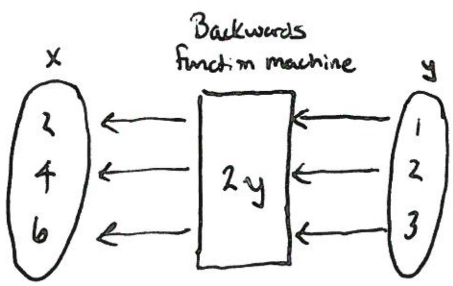
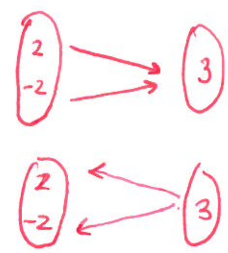
## Inverse Functions

- Idea want a function that takes the output of a function and gives the input
- "Go backwards"




Function must be 1-1:




Otherwise inverse won't be a function!

**Definition** - Let $f(x)$ be a 1-1 function then its <u>inverse</u>, $f^{-1}(x)$, satisfies:

$$\boxed{y = f(x) \iff f^{-1}(y) = x}$$

To Find the Inverse Function:

- swap x and y values
- solve for y

OR

- solve for x
- swap x and y values

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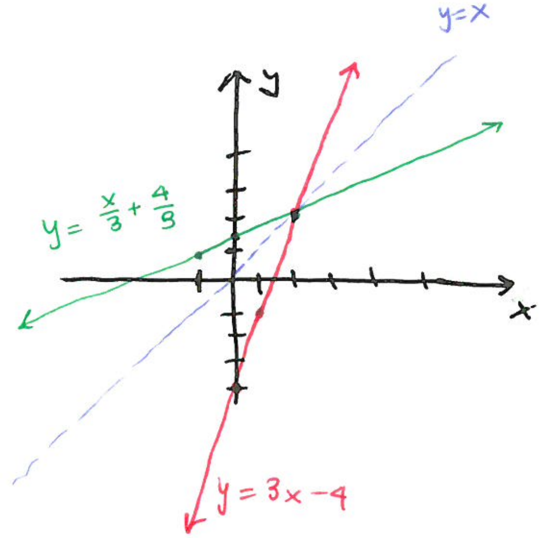
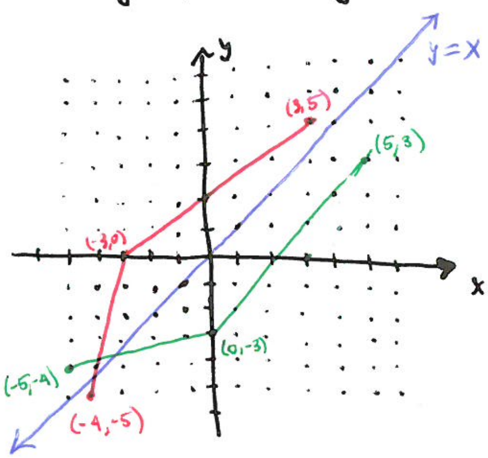
Ex. Find the inverse function of $y = 3x - 4$

- Swap x and y: $x = 3y - 4$
- Solve for y: $\boxed{y = \frac{x}{3} + \frac{4}{3}}$

$\boxed{\text{Graphically reflect } f(x) \text{ about the line } x=y}$




Ex. Sketch the graph of $g'(x)$ where $g(x)$ is given below:




Ex. Find the inverse of $y = f(x) = \frac{2}{3-x}$

- Swap x and y: $x = \frac{2}{3-y}$
- Solve for y: $3 - y = \frac{2}{x}$

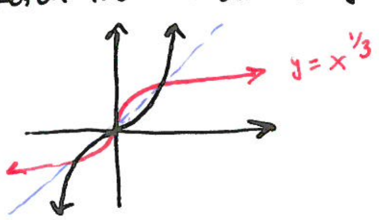
$y = 3 - \frac{2}{x}$ So $\boxed{f^{-1}(x) = 3 - \frac{2}{x}}$

$\boxed{\text{Your turn}}$

① Find the inverse of $h(x) = -\frac{5}{x} + 7$

$\boxed{h^{-1}(x) = \frac{-5}{x-7}}$

② Sketch the inverse of $y = x^3$ using the graph of $y = x^3$.



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**Lesson 32**

9/28/15

Agenda: 9/28/15

| | Period 2 | Period 8 |
|---|---|---|
| HW leader: | Tammon K | Stephanie D. |

lesson 32 part 2

Inverse Trigonometric functions

★ Handout Test 4 study Guide

Test 4 on Wednesday 1-30

- 90% from 21-30
- 14 problems
- fill in Unit Circle
- functions same as quiz
- overlapping triangles
- rate problems
- log/exp problems
- trig evaluation
- sign of trig functions
- related angles

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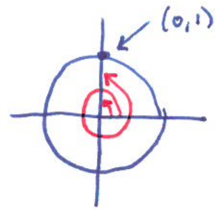
- Give all angles $\theta$ such that $\sin\theta = 1$

$\theta = 90°,\ 90°+360°,\ 90°+360°k$
for k any integer




Trig Functions
- input Angles
- output #

Inverse Trig Functions
- input #
- output Angles

★ So Sine is not a 1-1 function for all angles
BUT we want an inverse function so bad!

To Fix this we restrict the sine function to the values of $\theta$ between $-90°$ and $90°$ so that each output has exactly one input to have an inverse for these angles.

Definition - We define the inverse of sine or $\sin^{-1}(x)$ or $\arcsin(x)$ such that

$$y = \sin(\theta) \iff \theta = \sin^{-1}(y) \quad \theta = \arcsin(y)$$

only for $\boxed{-90° \le \theta \le 90°}$ [Same for $\tan^{-1}(x)$]

Example: Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$

$\sin\theta = \frac{\sqrt{3}}{2}$ so $\boxed{\theta = 60°}$

Evaluate $\arctan(-1) = \theta$

$\tan\theta = -1$ so $\boxed{\theta = -45°}$

Definition - We define the inverse of cosine, $\cos^{-1}(x)$ or $\arcsin(x)$ such that

$$y = \cos(\theta) \iff \theta = \cos^{-1}(y) \text{ or } \theta = \arccos(x)$$

Only for $\boxed{0 \le \theta \le 180°}$

Pre-Calc AB

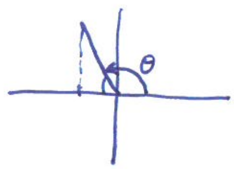
# Lesson 32

9/28/15

★ Inverse Trig functions only output one angle

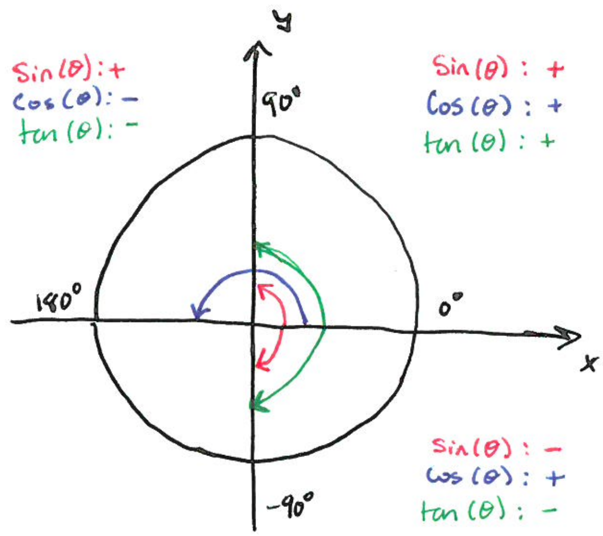
★ But solving Trig equations can have many solutions

**Ex.** Evaluate $\arccos\left(-\frac{1}{2}\right) = \theta$

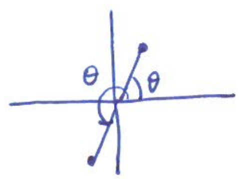



$\cos\theta = -\frac{1}{2}$

$\boxed{\theta = 120^\circ}$



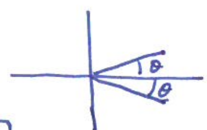

**Ex.** Solve $\tan\theta = \sqrt{3}$




$\theta = 60^\circ, 60^\circ + 180^\circ, 60 - 180^\circ \ldots$

$\boxed{\theta = 60^\circ + 180^\circ k \text{ for any integer } k}$

**Ex.** Solve $\cos\theta = \frac{\sqrt{3}}{2}$

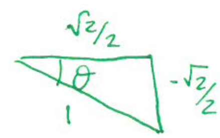



$\theta = 30^\circ + 360^\circ k$

or   $k$ any integer

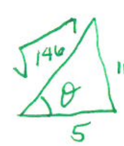
$\theta = -30^\circ + 360^\circ k$

**Ex.** Evaluate $\tan\left(\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right)$   $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = \theta$




$= \tan(\theta) = \dfrac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{1}$

**Ex.** Evaluate $\cos\left(\arctan\left(\frac{11}{5}\right)\right)$   $\arctan\left(\frac{11}{5}\right) = \theta$




$= \cos(\theta) = \boxed{\dfrac{5}{\sqrt{146}}}$

Pre-Calc AB

# Lesson 33

9/29/15

**Agenda:** 9/29/15

Period 2 — Period 8

HW leader: Emily H.

lesson 33
- Quadrilaterals
- Parallelograms
- Trapezoids

★ Test 4 tomorrow

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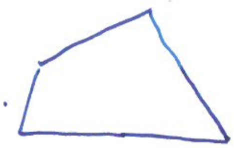
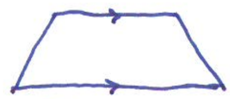
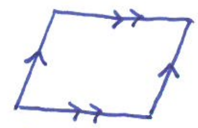
A **Quadrilateral** is a 4-sided polygon.

1. No sides are parrallel
2. 2 sides are parallel
3. 2 pairs of parallel sides

1. 

2. 

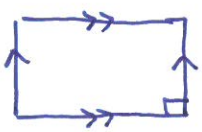
Trapezoid

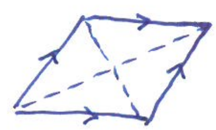
3. 

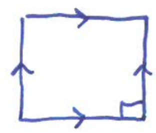
Parallelogram

**Types of Parallelograms:**

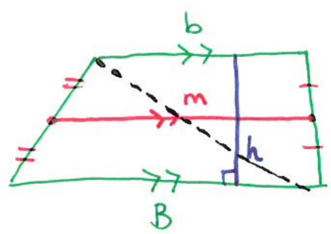
1. Rectangle - one 90° angle
2. Rhombus - 2 consecutive sides congruent
3. Square - both a rectangle and a rhombus

1. 

2. 

3. 

**Trapezoids**: Exactly one pair of sides are parallel



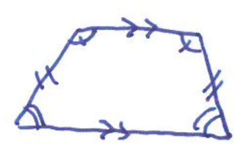

- Parallel sides - bases
- Non-parallel sides - legs
- Median line segment
- Altitude of a trapezoid

**Properties:**

1. Area of Trapezoid $= \frac{1}{2}h(B+b)$
2. The median is parralel to the bases.
3. $m = \frac{1}{2}(B+b)$

**Isoceles Trapezoids:**

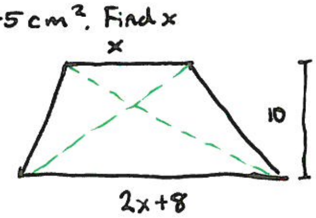
Legs are congruent



**Properties:**

1. lower base angles are congruent
2. upper base angles are congruent
3. The diagonals are congruent
4. lower and upper base angles are supplementary

**Ex. 33.6** The trapezoid's area is $175\text{ cm}^2$. Find x




$175 = \frac{1}{2}(10)(x + 2x + 8)$

$35 = 3x + 8$

$\boxed{x = 9\text{ cm}}$

Pre-Calc AB

# Lesson 34

10/1/15

Agenda: 10/1/15

Period 2

Period 8

HW leader:

Lesson 34

Summation Notation

Linear Regression

Decomposing functions

Test back after lesson

## Summation Notation

$$\sum_{i=1}^{4} i^2$$

end value

Typical element

start value

variable of summation

$$\sum_{i=1}^{4} i^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2 = 30$$

★ Any symbol other than the variable of the summation is assumed to be constant

Ex: $\sum_{y=0}^{5} k = \underset{0}{k} + \underset{1}{k} + \underset{2}{k} + \underset{3}{k} + \underset{4}{k} + \underset{5}{k} = \boxed{6k}$

Ex 34.3 Evaluate: $\sum_{j=0}^{3} \frac{2^j}{j+1} = \frac{2^0}{0+1} + \frac{2^1}{1+1} + \frac{2^2}{2+1} + \frac{2^3}{3+1}$

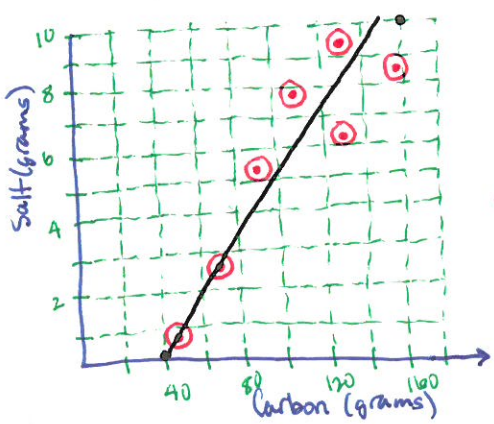
$= 1 + 1 + \frac{4}{3} + 2 = \boxed{\frac{16}{3}}$

## Linear Regression

Finding a linear equation that best fits data
Sometimes the data is scattered so they may not all fall on the line that best models their relationship.

Ex. 34.5 Write the equation of the line that gives Salt as a function of Carbon ($S = mC + b$)

| Carbon | Salt |
|---|---|
| 48 | 0.8 |
| 70 | 2.6 |
| 90 | 5.4 |
| 108 | 7.6 |
| 130 | 6.3 |
| 132 | 9.4 |
| 156 | 8.4 |




Need two points on the line:

(40, 0) and (160, 10)

Slope $m = \frac{10-0}{160-40} = \frac{10}{120} = \frac{1}{12}$

b: $0 = \frac{1}{12}(40) + b$ so $b = -\frac{10}{3}$

$$\boxed{S = \frac{1}{12}C - \frac{10}{3}}$$

Pre-Calc AB

# Lesson 34

10/1/15

## Decomposing Functions: Undoing a Composition

Ex. Find two functions such that:

$f \circ g(x) = \sqrt[3]{x^2-1}$ : $g(x) = x^2-1$ $f(x) = \sqrt[3]{x}$

$f \circ g(x) = 2^{-x}$ : $g(x) = -x$ $f(x) = 2^x$

$f \circ g(x) = \ln|x+3|$ : $g(x) = |x+3|$ $f(x) = \ln(x)$

$f \circ g(x) = \dfrac{3}{(x-4)^2}$ : $g(x) = x-4$ $f(x) = \dfrac{3}{x^2}$

Pre - Calc AB

**Lesson 35**

10/2/15

Agenda: 10/2/15

Period 2

Period 8

HW leader:

lesson 35

Change in Coords

Distance formula

Name of a number

☆ Will need a graphing calculator after Fall break.

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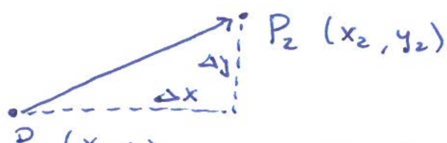
## Change in Coordinates:




$\Delta x = x_2 - x_1$ Change in the x-coord from $P_1$ to $P_2$

$\Delta y = y_2 - y_1$ Change in the y-coord from $P_1$ to $P_2$

Ex. Given $P_1\ (-5, 1)$ and $P_2\ (-3, -7)$ find the change in coordinates.

$\Delta x = x_2 - x_1 = -3 - (-5) = \boxed{2}$

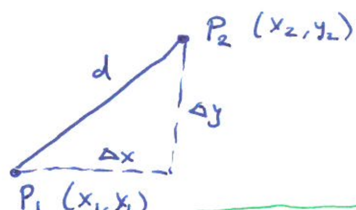
$\Delta y = y_2 - y_1 = -7 - (1) = \boxed{-8}$

## Distance Formula:

Distance between two points, just Pythagorean Theorem

$d = \sqrt{\Delta x^2 + \Delta y^2}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

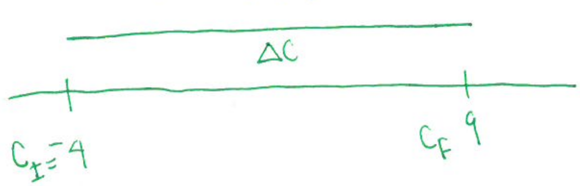



Ex. Find the distance between $(2, -3)$ and $(x, y)$

$\boxed{d = \sqrt{(x-2)^2 + (y+3)^2}}$

## Name of a Number:

Ex. 35.4 What is the number that is $\frac{3}{7}$ of the way from $-4$ to $+9$?




$\Delta C = C_F - C_I = 9 - (-4) = 13$

$\text{Number} = C_I + \Delta C \cdot \frac{3}{7}$

$= -4 + 13 \cdot \frac{3}{7}$

$= \boxed{\frac{11}{7}}$