Pre-Calc AB

Lesson 21

9/3/15

Agenda: 9/3/15

HW leader:

Period 2	Period 8
Sydney W.	Leah C.

lesson 21

Evaluating Functions

Domain and Range

Tests for functions

★ Test 2 Back after lesson

★ Definition - A function is a mapping, such that for each input there is exactly one output.

Set of inputs - Domain

Set of outputs - Range

Write $y = f(x)$

independent variable

Denotes a function of x

dependent variable

Evaluating functions:

- Given an input what is the output

Ex. Consider $f(x) = x^2 - x$

Evaluate

$$f(5) = (5)^2 - (5) = 25 - 5 = \boxed{20}$$

$$f(x+2) = (x+2)^2 - (x+2) = x^2 + 4x + 4 - x - 2$$
$$= \boxed{x^2 + 3x + 2}$$

$$3f(x+h) = 3[(x+h)^2 - (x+h)] = 3[x^2 + 2xh + h^2 - x - h]$$
$$= \boxed{3x^2 + 6xh + 3h^2 - 3x - 3h}$$

Pre-Calc AB

Lesson 21

9/3/15

Domain – What is all the allowable input to the function

Range – What is all possible outputs given all possible inputs

★ **For domain**: No inputs allowed if we then divide by zero for have a negative number in an even radical.

Ex 21.6 Find the domain and range of $f(x) = \sqrt{x}$.

Need $x \geq 0$ for input so

x can be all real numbers with $x \geq 0$

Notation:

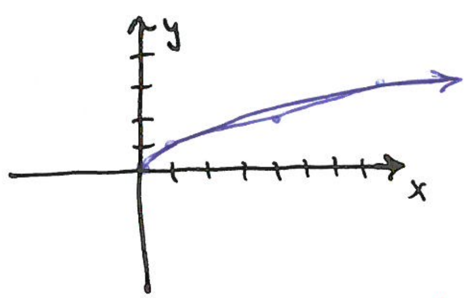
set of, in, such that

$$\text{Domain of } f = \{x \in \mathbb{R} \mid x \geq 0\}$$ [Set-builder Notation]

$$= [0, \infty)$$ [Interval notation]

$[$ means inclusion; $)$ can't contain infinity

x	$f(x)$
0	0
1	1
4	2
9	3
16	4



Graphing all pairs (x, y) Gives the graph of $f(x)$.

See that y-values increase as x-values increase so

$$\text{Range of } f = \{y \in \mathbb{R} \mid y \geq 0\} = [0, \infty)$$

Ex. Find the domain of $f(x) = \dfrac{\sqrt{x}}{(x+5)(x-1)}$

- $x \neq -5, 1$
- $x \geq 0$

$$\text{Domain of } f = \{x \in \mathbb{R} \mid x \geq 0,\ x \neq 1\}$$

$$= [0, 1) \cup (1, \infty)$$

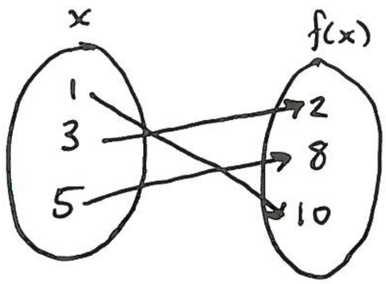
up to but not including 1

Pre-Calc AB

Lesson 21

9/3/15

Representations of functions



mapping

$\{(1,10),(3,2),(5,8)\}$

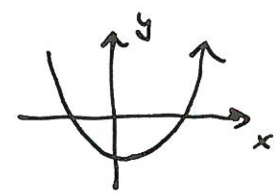
Set of ordered pairs

x	1	2	4	8
f(x)	-5	6	7	6

table

$y = f(x) = x^2 - 4$

equation



graph

Tests For Functions

- Equation - Can you solve for the dependent variable in terms of the independent
- map, table, set - for every input is there exactly one output
- Graph - Every vertical line intersects the graph at most once.

Ex. Which are or are not functions? Explain why or why not.

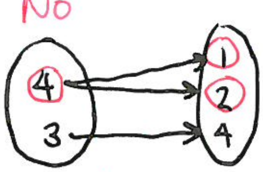
① Yes

x	-1	0	1	4
f(x)	2	0	2	6

② No

$\{\boxed{(1,3)},(2,-3),(4,2),\boxed{(1,4)},(5,2)\}$

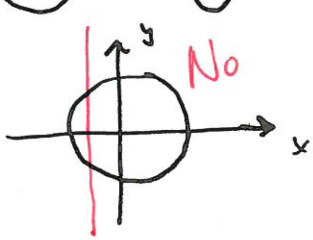
③ No



④ No $y = \pm\sqrt{3x^3+2}$

$y^2 - 2 = 3x^3$ [is y a function of x?]

⑤ No



⑥ Yes

One-to One function

every output has exactly one input - horizontal line test.

Pre-Calc AB | **Lesson 21** | 9/4/15

Agenda: 9/4/15 — Period 2 — Period 8

HW leader: None

Lesson 21

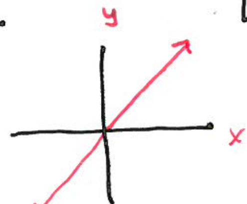
- Graphs
- Transformations
- Piecewise functions

☆ Will be having non-calculator portions of tests

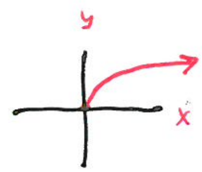
Basic Graphs

Must Know these

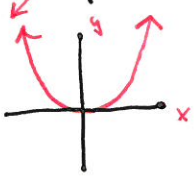
$f(x) = x$



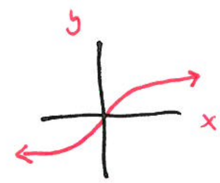
$f(x) = \sqrt{x}$



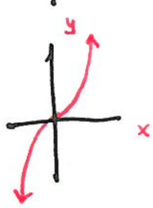
$f(x) = x^2$



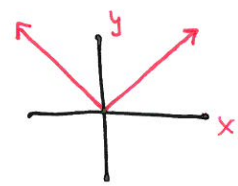
$f(x) = \sqrt[3]{x}$



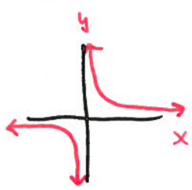
$f(x) = x^3$



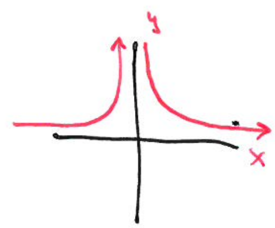
$f(x) = |x|$



$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x^2}$

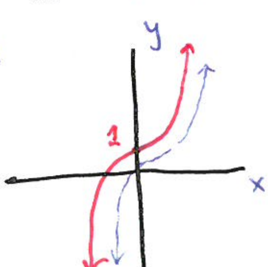


Function Transformations

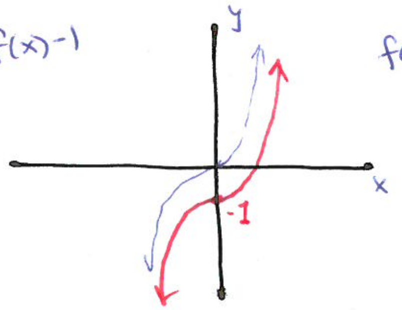
$f(x) = x^3$

- Vertical shifts: $f(x) + K$ — K a real number
  - $K > 0$ go up K units
  - $K < 0$ go down K units
- Horizontal shifts: $f(x + K)$ — K a real number
  - $K > 0$ go left K units
  - $K < 0$ go right K units

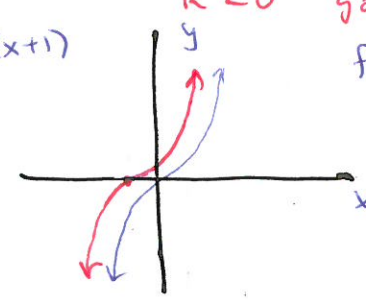
$f(x)+1$



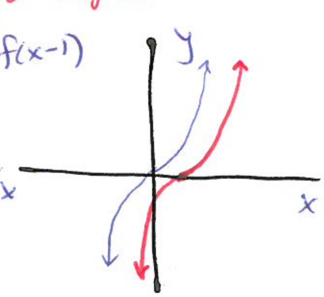
$f(x)-1$



$f(x+1)$



$f(x-1)$

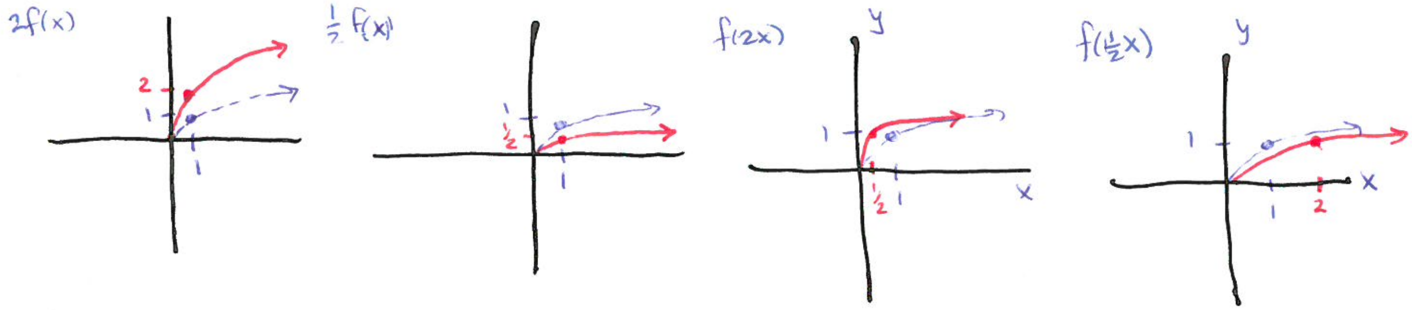


Pre-Calc AB

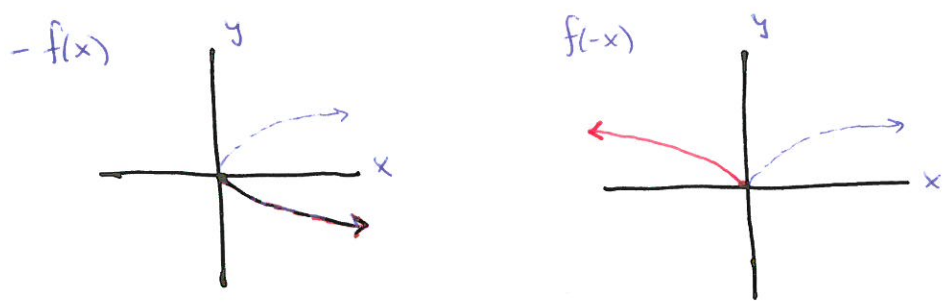
Lesson 21

9/5/15

- Vertical stretch: $k \cdot f(x)$, $k > 1$ $\qquad f(x) = \sqrt{x}$
- vertical shrink: $k \cdot f(x)$, $k < 1$
- Horizontal stretch: $f(kx)$, $k < 1$
- Horizontal shrink: $f(kx)$, $k > 1$



- reflection about x-axis: $-f(x)$
- reflection about y-axis: $f(-x)$



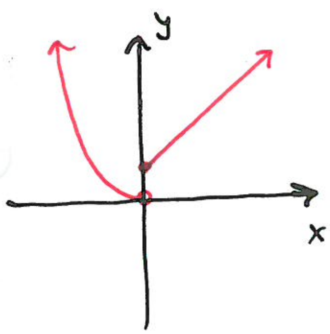
Piecewise Functions - defined in pieces

Ex.

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$$

function to use when input is < 0

function to use when input is ≥ 0



$f(2) = 2+1 = 3$

$f(0) = 0+1 = 1$

$f(-2) = (-2)^2 = 4$

- X-intercepts (horizontal intercepts) where the graph hits the x-axis [when $y = 0$]
- y-intercept (vertical intercept) where the graph hits the y-axis [when $x = 0$]

Pre-Calc AB

Lesson 22

9/8/15

Agenda: 9/8/15

- Lesson 21/22
  - transformations
  - Piecewise / One-to-One
  - Absolute Value
- ★ Quiz 3 tomorrow lessons 16-21

Use as a Study Guide

★ WS 4,5 Due tomorrow

★ HW 22 Due tomorrow

P2
P8

- Transformations finish

Work on WS

Order of transformations: $\boxed{\text{In to Out}}$

$y = cf(a(x+b)) + d$

Ex. list the transformations of $g(x) = -\frac{1}{5}f(2x+4)+3$ from $f(x)$

$$g(x) = -\frac{1}{5}f(2(x+2)) + 3$$

- If $(4,10)$ is a point on $f(x)$ then what point must be on $g(x)$?

$(2,10)$	• Horizontal shrink by 2
$(0,10)$	• Horizontal shift left by 2
$(0,2)$	• Vertical shrink by 5
$(0,-2)$	• reflect about the x-axis
$\boxed{(0,1)}$	• shift up 3

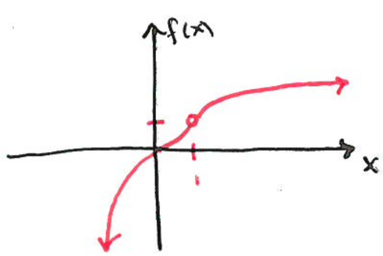
Check:

$$g(0) = -\frac{1}{5}f(2(0+2)) + 3$$
$$= -\frac{1}{5}f(4) + 3$$
$$= -\frac{1}{5}(10) + 3$$
$$= -2 + 3$$
$$= -1 \checkmark$$

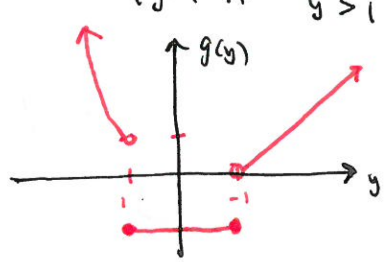
★ Work on # 16-17

P2 Piecewise Functions:

Graph: $f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x > 1 \\ x^3 & \text{if } x < 1 \end{cases}$



Graph: $g(y) = \begin{cases} y^2 & \text{if } y < -1 \\ -1 & \text{if } -1 \le y \le 1 \\ y-1 & \text{if } y > 1 \end{cases}$



Pre-Calc AB

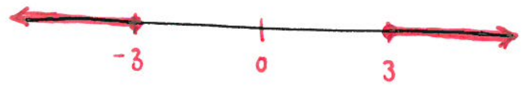
Lesson 22

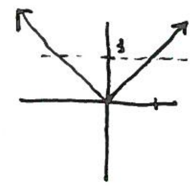
9/8/15

<u>Absolute Value:</u>

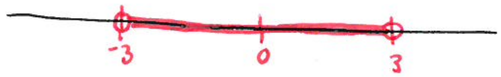
Graph on a number line:

- $|x| \geq 3$



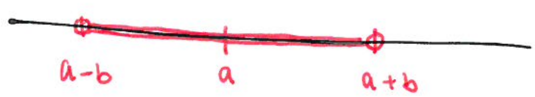


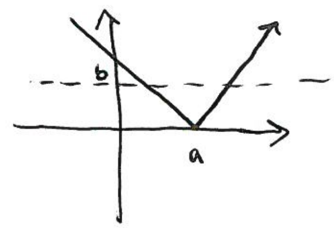
- $|x| < 3$



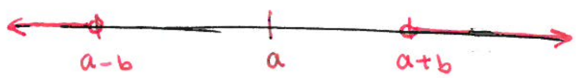
- $|x - 7| > 3$

- $|x - a| < b$





- $|x - a| > b$



Pre-Calc AB

Lesson 23

9/10/15

Ray X.

Michael C.

Agenda: 9/10/15

- lesson 23
  - exponential functions
  - Sketching Exponentials

★ Begin working on HW 23

★ Quiz B back after lesson

Exponential Functions

- Properties of exponents work for any real number → exponential functions

Definition - If $a > 0$, $a \neq 1$ then $f(x) = a^x$ is the exponential function with base a.

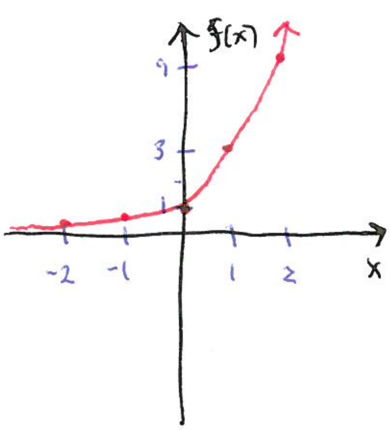
[$a = 0$ or $a = 1$ are just constant functions]

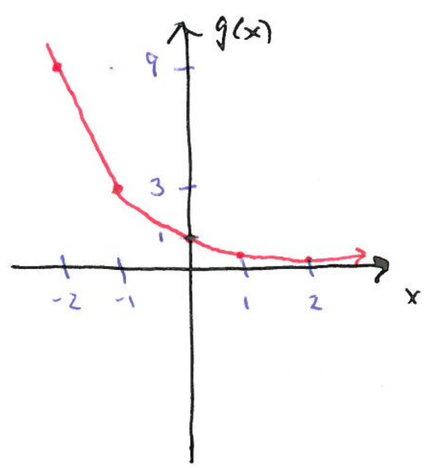
Properties of exponents: $a > 0$, $a \neq 1$

1. a^x is a unique real number for all real numbers x
2. $a^b = a^c$ if and only if $b = c$
3. If $a > 1$ and $m < n$ then $a^m < a^n$
4. If $0 < a < 1$ and $m < n$ then $a^m > a^n$

Ex. sketch the graph of $f(x) = 3^x$ and $g(x) = \left(\frac{1}{3}\right)^x$

x	$f(x)$
-2	1/9
-1	1/3
0	1
1	3
2	9





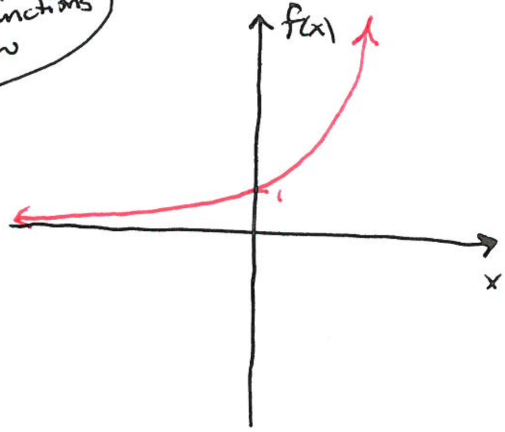
x	$g(x)$
-2	9
-1	3
0	1
1	1/3
2	1/9

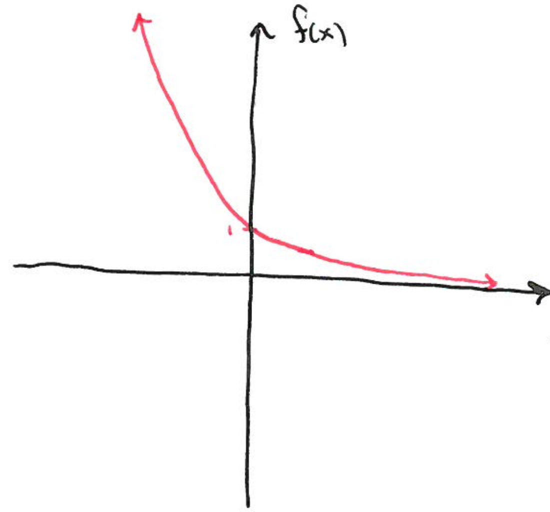
Pre-Calc AB

Lesson 23

9/10/15

★ Add to list of functions to know





$f(x) = a^x, \quad a > 1$

Domain: $\mathbb{R}$

Range: $(0, \infty)$

$f(x) = a^x, \quad 0 < a < 1$

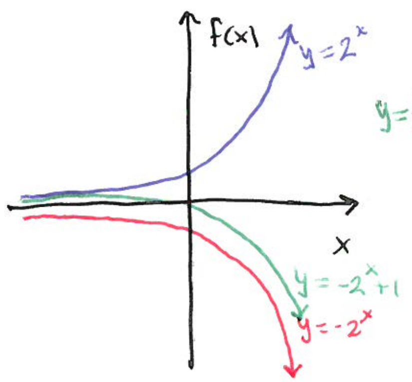
$\left[f(x) = \left(\frac{1}{a}\right)^{-x} \right]$

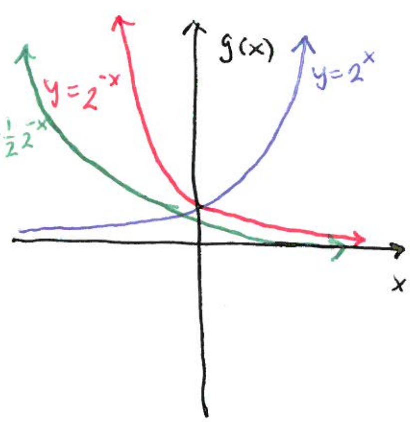
Ex. Graph the following:

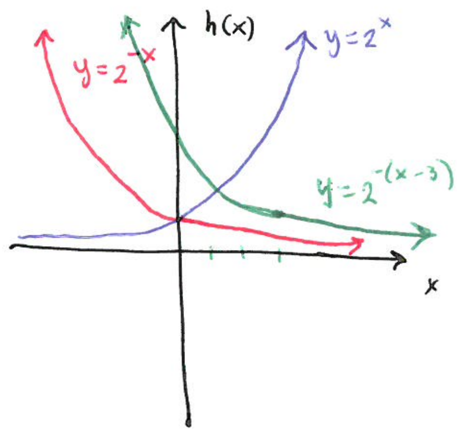
$f(x) = -2^x + 1$

$g(x) = \frac{1}{2} 2^{-x}$

$h(x) = 2^{-x+3} = 2^{-(x-3)}$







Pre-Calc AB

Lesson 24

9/11/15

Agenda: 9/11/15

HW leader: Period 2: Catherine X. Period 8: Sophia K.

lesson 24

Combining functions

★ Test 3 on Wednesday

Let f and g be two functions.

New Functions: **Domain**

- Sum $(f+g)(x) = f(x) + g(x)$
- Difference $(f-g)(x) = f(x) - g(x)$
- Product $(f \cdot g)(x) = f(x) \cdot g(x)$

} (Domain of f) $\cap$ (Domain of g)

- Quotient $(f/g)(x) = \dfrac{f(x)}{g(x)}$

} And $g(x) \neq 0$

Ex. $f(x) = x + 2$ $g(x) = 3x^2$

$(f+g)(x) = f(x) + g(x) = (x+2) + (3x^2) = 3x^2 + x + 2$

$(f \cdot g)(x) = f(x) \cdot g(x) = (x+2)(3x^2) = 3x^3 + 6x^2$

} Domain: $\mathbb{R}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{3x^2}{x+2}$ Domain: $(-\infty, -2) \cup (-2, \infty)$

$\{x \in \mathbb{R} \mid x \neq -2\}$

★ Function Composition $f \circ g(x) = f[g(x)]$ [or function inception]

$$x \longrightarrow \underset{}{\overset{g(\ )}{\boxed{}}} \xrightarrow{g(x)} \overset{f(x)}{\boxed{}} \longrightarrow f[g(x)]$$

[plugging in a function to a function]

x. $f(x) = x^2 - 2$ and $g(x) = \sqrt{x}$

★ Domain: Don't simplify

$f \circ g(x) = f[g(x)] = f(\sqrt{x}) = (\sqrt{x})^2 - 2$ Domain: $[0, \infty)$

$g \circ f(x) = g[f(x)] = g(x^2 - 2) = \sqrt{x^2 - 2}$ Domain: $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$f \circ g(6) = f(\sqrt{6}) = (\sqrt{6})^2 - 2 = \boxed{4}$

Pre-Calc AB

$\boxed{\text{Lesson 24}}$

9/11/15

Ex: $h(z) = 2^z$ $\quad w(z) = \frac{\sqrt{z}}{z-4}$ Find $h \circ w(z)$, $w \circ h(z)$ and their domains

$h \circ w(z) = h[w(z)] = h\left(\frac{\sqrt{z}}{z-4}\right) = 2^{\frac{\sqrt{z}}{z-4}}$

$z \geq 0 \quad z \neq 4$

$\boxed{\text{Domain: } [0,4) \cup (4,\infty)}$

$w \circ h(z) = w[h(z)] = w(2^z) = \frac{\sqrt{2^z}}{2^z - 4}$

$2^z \geq 0$ ✓ $\quad 2^z \neq 4$ so $z \neq 2$

$\boxed{\text{Domain: } (-\infty,2) \cup (2,\infty)}$

$h \circ w(3) = h(w(3)) = h(\sqrt{3}) = 2^{-\sqrt{3}} = \boxed{\frac{1}{2^{\sqrt{3}}}}$

$w \circ h(-1) = w(h(-1)) = w(2^{-1}) = \frac{\sqrt{2^{-1}}}{2^{-1} - 4} = \frac{\frac{1}{\sqrt{2}}}{-\frac{7}{2}} = \frac{-2}{7\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{7}}$

Pre-Calc AB

Lesson 25

9/14/15

Agenda: 9/14/15

HW leader: Period 2 Ava U. Period 8 Emma T.

Lesson 25

Rate Problems

★ Handout WS 6

★ Test 3 on Wednesday
Lessons 1-22

Ex: One pump can fill a pool with volume 2000 gallons in 5 hours and another pump can fill the same pool in 4.5 hours. Together how long will it take to fill the pool?

	rate	time	job
Pump 1	$1/5$	5hr	1
Pump 2	$1/4.5$	4.5hr	1
Together	$\frac{1}{5}+\frac{1}{4.5}$	T	1

$$\left(\frac{1}{5}+\frac{2}{9}\right)T = 1$$

$$T = 1\left(\frac{9+10}{45}\right)^{-1} = \frac{45}{19} \approx 2.37 \text{ hours}$$

How long would it take to fill a pool with volume 2743 gallons?

1	$2000/5$	5hr	2000
2	$2000/4.5$	4.5hr	2000
1+2	$2000\left(\frac{19}{45}\right)$	T	2743

$$2000\left(\frac{19}{45}\right)T = 2743$$

$$T = \frac{2743 \cdot 45}{2000 \cdot 19} \approx 3.25 \text{ hours}$$

Ex: If 10 workers can build a house in 45 days, how many workers are needed to build 3 houses in 70 days? Round to the whole worker :)

rate per worker	workers	time	jobs
$1/10\cdot45$	10	45	1
$3/70\cdot w$	W	70	3

$$\frac{1}{10\cdot45} = \frac{3}{70\cdot w}$$

$$w = \frac{30\cdot45}{70} = \frac{3\cdot45}{7}$$

$$\approx 19.29$$

$\boxed{\text{Will need 20 workers}}$

Ex: 232 pancakes feeds 58 hungry students for 4 days. When 10 more Students are added and 150 more pancakes are provided, how long will the pancakes last? Round to the whole day.

rate	Students	days	Pancakes
$232/58\cdot 4$	58	4	232
$282/68\cdot T$	68	T	

$$\frac{232}{58\cdot 4} = \frac{382}{68\cdot T}$$

$$T = \frac{58\cdot 4\cdot 382}{68\cdot 232} \approx 5.62$$

$\boxed{\text{The pancakes will last about 5 days.}}$