Pre-Calc AB

Lesson 16

8/25/15

Agenda: 8/25/15

HW Leader:

Lesson 16

Complex Fractions

Abstract Equations

Division of polynomials

Period 2: McKenna P.

Period 8: Hanah B.

* Quiz 3 tomorrow

Complex Fractions (i.e. what I called stacked fractions)

Ex 16.1 Simplify $\dfrac{x}{a + \dfrac{m}{b + \frac{c}{d}}}$

$$= \frac{x}{a + \dfrac{m}{\frac{bd+c}{d}}} = \frac{x}{a + \frac{dm}{bd+c}}$$

$$= \boxed{\frac{(bd+c)x}{a(bd+c)+dm}}$$

Abstract Equations (No numbers; My Favorite!)

Ex. Solve $x = \frac{c}{z}\left(a + \frac{b}{my}\right)$ for m.

$$\Rightarrow \frac{zx}{c} = a + \frac{b}{my} \Rightarrow \frac{zx}{c} - a = \frac{b}{my}$$

$$\Rightarrow \frac{1}{y} = \frac{m}{b}\left(\frac{zx}{c} - a\right) \Rightarrow y = \frac{1}{\frac{m}{b}\left(\frac{zx}{c} - a\right)}$$

$$\Rightarrow \boxed{y = \frac{bc}{m(zx - ac)}}$$

Pre-Calc AB

Lesson 16

8/25/15

Division of Polynomials

Ex. Divide x^3+1 by $2x-1$

$$\begin{array}{r|l} & \frac{1}{2}x^2+\frac{1}{4}x+\frac{1}{8} \\ \hline 2x-1 & 1x^3+0x^2+0x+1 \\ & -1x^3+\frac{1}{2}x^2 \\ \hline & 0\quad \frac{1}{2}x^2+0x+1 \\ & -\frac{1}{2}x^2+\frac{1}{4}x \\ \hline & \frac{1}{4}x+1 \\ & -\frac{1}{4}x+\frac{1}{8} \\ \hline & \frac{9}{8} \end{array}$$

$$\frac{x^3+1}{2x-1} = \frac{1}{2}x^2+\frac{1}{4}x+\frac{1}{8}+\frac{9}{8(2x-1)}$$

Depending on Time:

$$\begin{array}{r|l} & x^2-x+3 \\ \hline x+1 & x^3+2x-1 \\ & -x^3+x^2 \\ \hline & -x^2+2x-1 \\ & +x^2+x \\ \hline & 3x-1 \\ & -3x-3 \\ \hline & -4 \end{array}$$

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Agenda: 8/27/15

HW Leader: None

Lesson 17

Proofs of Pythagorean Thm

Proofs of Similarity

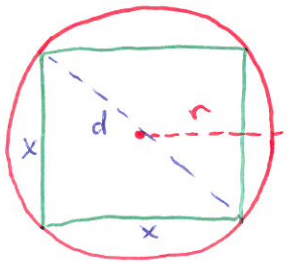
Warm-Up:

① A square lawn has area 800 ft². A sprinkler placed at the center of the lawn sprays water in a circular pattern that just covers the lawn. What is the radius of the circle?

$\text{Area}_{\square} = x^2 = 800$

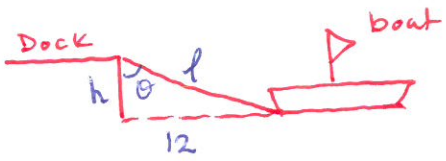
So $x = \sqrt{800} = 20\sqrt{2}$ ft

$d = \sqrt{2}x = 40$ ft $\Rightarrow$ $\boxed{r = 20\text{ ft}}$



② A boat with a rope attached at water level is being pulled into a dock. When the boat is 12 ft from the dock, the length of the rope is 3 ft more than twice the height of the dock above water.

- Find the height of the dock
- Find the angle of depression from the dock to the boat. to the thousandths.



$l = 2h + 3$

$l^2 = h^2 + 12^2$ So

$(2h+3)^2 - h^2 = 144$

$4h^2 + 12h + 9 - h^2 = 144$

$3h^2 + 12h - 135 = 0$

$h^2 + 4h - 45 = 0$

$(h+9)(h-5) = 0$

So $h = -9$ or $\boxed{h = 5\text{ ft}}$

Height of dock is 5 ft when boat is 12 ft away.

$\tan\theta = \frac{12}{h} = \frac{12}{5}$

So $\theta = \tan^{-1}\left(\frac{12}{5}\right) \approx \boxed{67.380^\circ}$

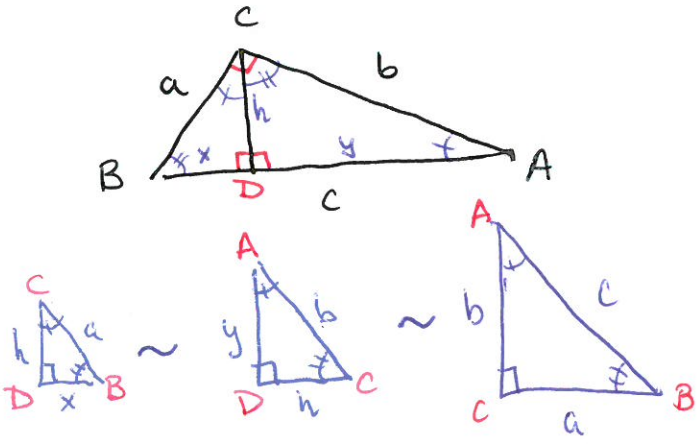
Pre-Calc AB

Lesson 17

8/27/15

Proofs of the Pythagorean Theorem

Proof (1)



Since $\triangle DBC \sim \triangle CBA$. Then:

$$\frac{c}{a} = \frac{b}{h} = \frac{a}{x} \quad \Rightarrow \quad cx = a^2$$

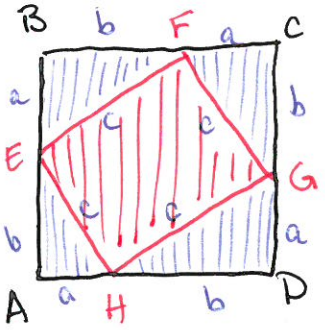
Since $\triangle DCA \sim \triangle CBA$

$$\frac{c}{b} = \frac{b}{y} = \frac{a}{h} \quad \Rightarrow \quad cy = b^2$$

Thus $a^2 + b^2 = cx + cy$

$= c(x+y) = c^2$ ∎

Proof (2)



Area of Shaded □

= Area of big □ − Area of 4 Triangles

$c^2 = (a+b)^2 - 4\left(\frac{1}{2}a \cdot b\right)$

$c^2 = a^2 + 2ab + b^2 - 2ab$

$$\boxed{c^2 = a^2 + b^2}$$ ∎

Pre-Calc AB

Lesson 17

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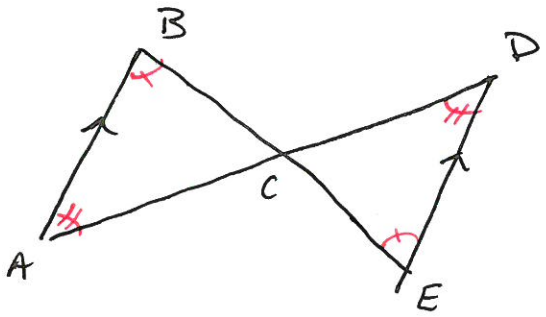
★ To prove two △ are similar, prove congruent or all angles are equal.

Ex. 17.2 Prove $\triangle ABC \sim \triangle DEC$



Statements	Reasons
1. $\overline{BA} \parallel \overline{DE}$	Given
2. $\angle ACB \cong \angle DCE$	Vertical opposite Angles
3. $\angle B \cong \angle E$	} Alternate interior angles of parallel lines
4. $\angle A \cong \angle D$	
5. $\triangle ABC \sim \triangle DEC$	AAA

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Agenda: 8/28/15

HW leader:

Lesson 18

Word Problems

★ Test 2 on Wednesday

Period 2: Nate G.

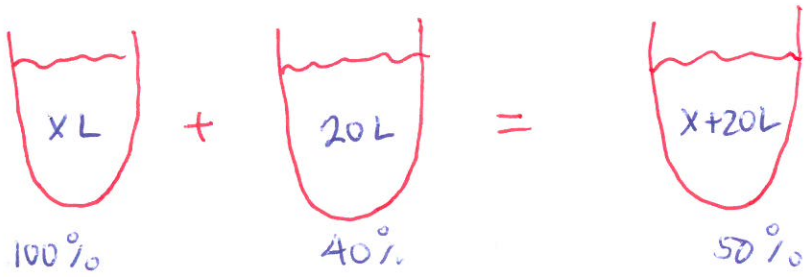
Period 8: Joseph P.

Strategy for Word Problems

1. Read the Problem, assign variables to what you need to find.
2. Draw a Picture and label unknowns.
3. Write equations for what's given and needs to be found,
4. Use the equations to solve for what you want
5. Check! Is my answer reasonable? Does it make sense?

(1) How much pure alcohol should be added to 20 L of 40% alcohol to increase the concentration to 50% alcohol?

Let x = amount of pure alcohol to be added



$$1 \cdot x + 0.4(20) = 0.5(x+20)$$

$$x + 8 = 0.5x + 10$$

$$0.5x = 2$$

$$\boxed{x = 4 \text{ Liters}}$$

② A cup of rice contains 5 grams of protein and 205 calories and a cup of soybeans contains 68 grams of protein and 830 calories. How many cups of each should be used for a meal containing 20.5 grams of protein and 351 calories?

	rice	soybeans	meal
	r cups	+ s cups	= r+s cups
Protein:	$\frac{5\text{ g}}{1\text{ cup}}$	$\frac{68\text{ g}}{1\text{ cup}}$	20.5 grams
Calories:	$\frac{205\text{ cal}}{1\text{ cup}}$	$\frac{830}{1\text{ cup}}$	351 cals

① $.5r + 68s = 20.5$

② $205r + 830s = 351$

Sub s into ①:

$r = 0.7$ cups

① * (-41): $-205r - 2788s = -840.5$

+ ②: $205r + 830s = 351$

$-1958s = -489.5$

$s = 1/4$ cups

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1. $V \approx 36.4\ ft^3$
2. 6 cm
3. w: 21 in l: 28 in
4. $96^2 = 2x^2 + -20x + 100$ $x = 40$ ft
5. a) $80 - x = u$
   b) $P = 400 + 3(x)$
   c) $R = u \cdot p = (80-x)(400+3x) = -20x^2 + 1200x + 32{,}000$
   d) 60 or 66 for x
6. a) $[0, 9]$
   b) $C(x) = 400(9-x) + 500\sqrt{36+x^2}$
   c) $C(6) \approx \$5442.62$
7. $APB = \sqrt{x^2+12^2} + \sqrt{(20-x)^2+16^2}$
   $l \approx 36.49$ ft.
8. 4 mL
9. 8 L

Challenge

14 x 7 by 6

10. $38 = \pi r^2 h$ or $38 = 2\pi r^2 + 2\pi rh$

$w = \frac{38}{\pi r^2}$

5.17 x 10.34 by 11

PRECALC AB PERIOD: _____ NAME: _____

Worksheet 3 - Lesson 18

Strategies for Word Problems

1. Read the problem carefully. Underline the important pieces of information. Assign variable names to the things you need or want to find.
2. Draw a picture! Label your variables in the picture.
3. Write equations for what's been given to you and for what you want to find.
4. Use the equations to solve for what you want.
5. CHECK YOUR ANSWER! Is my answer reasonable? Does it make sense? Does it have the correct units?

Together

1. How much pure alcohol should be added to 20 liters of 40% alcohol concentration to increase the concentration to 50% alcohol?

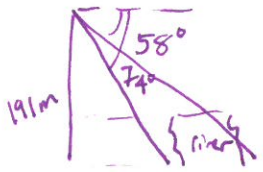
$x + 0.4(20) = 0.5(20+x)$

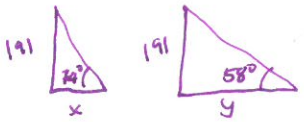
$x + 8 = 10 + \frac{1}{2}x$

$\frac{1}{2}x = 2$

$x = 4$ liters

2. From the top of a canyon, the angle of depression to the far side of the river is 58°, and the angle of depression to the near side of the river is 74°. The depth of the canyon is 191 meters. What is the width of the river at the bottom of the canyon? Round to the nearest tenth of a meter.





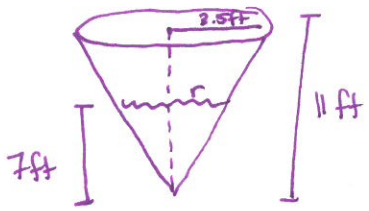
$x = \frac{191}{\tan 74°}$ $y = \frac{191}{\tan 58°}$

$y \approx 119.350$

$x \approx 54.768$

The width of the river is about 64.6 meters.

3. A grain bin in the shape of an inverted cone has height 11 feet and radius 3.5 feet. If the grain in the bin is 7 feet high, calculate the volume of the grain to one decimal place. *Hint:* Use similar triangles.

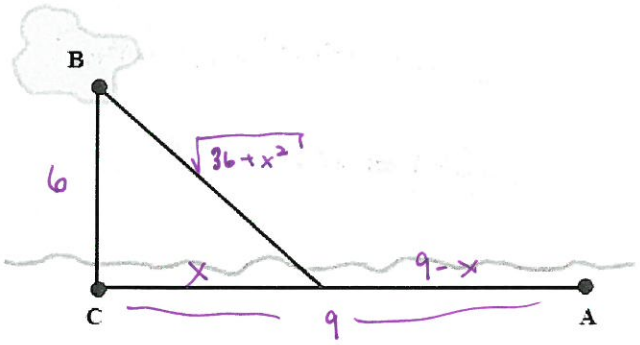


$\frac{3.5}{11} = \frac{r}{7} \Rightarrow r = \frac{24.5}{11} = \frac{49}{22}$

$\text{Volume} = \frac{(\pi r^2 \cdot h)}{3} = \frac{\pi\left(\frac{49}{22}\right)^2 \cdot 7}{3} \approx 36.4 \text{ ft}^3$

The volume of the grain is about 36.4 ft^3

4. A company wishes to run a utility cable from point A on the shore to an installation at point B on the island. The island is 6 miles from the shore and the company at A is 9 miles from point C. It costs \$400 per mile to run the cable on the land and \$500 per mile under water. Assume that the cable starts at A, runs along the shoreline, and then anlges and runs under water to the island. Let x represent the distance from C at which the under water portion of the cable run begins.



(a) What are the possible values for x?

$[0, 9]$

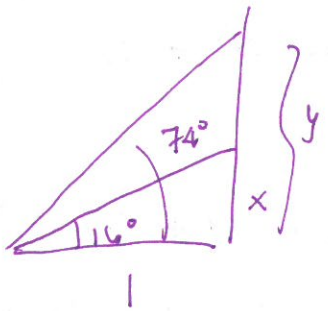
(b) Express the total cost of laying the cable in terms of x.

$C(x) = 400(9-x) + 500\sqrt{36+x^2}$

(c) Find the total cost if 3 miles of cable are on land.

$C(6) \approx \$5442.64$

5. Marion is observing the launch of a space shuttle from the command center. When she first sees the shuttle, the angle of elevation to it is 16°. About 10 seconds later, the angle of elevation is 74°. If the command center is 1 mile from the launch pad, how far did the shuttle travel while Marion was watching and what was it's average speed in miles per hour? Round to the nearest thousandth of a mile.



$x = \tan 16° \approx 0.287$ miles

$y = \tan 74° \approx 3.487$ miles

$d \approx 3.200$ miles

ave speed $\approx \frac{3.200}{10\text{s}} \cdot \frac{60\text{s}}{1\text{m}} \cdot \frac{60\text{m}}{1\text{hr}}$

≈ 1152.241 mph

Pre-Calc AB

Lesson 19

8/31/15

Agenda: 8/31/15

HW Leader:

Lesson 19

Period 2	Period 8
Nicole T.	Kim D.

Nonlinear Systems

Factoring Exponentials

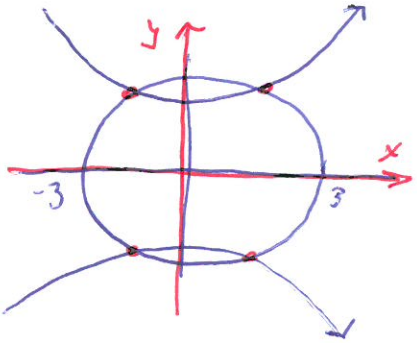
Sum and Difference of 2 cubes

★ Test 2 on Wednesday

Nonlinear Systems

Ex. 19.2 Solve (a) $x^2+y^2=9$ (circle)

(b) $2x^2-y^2=-6$ (hyperbola)



(a) + (b): $3x^2=3$ so $x=\pm 1$

- When $x=1$ then $y^2=8$ so $y=\pm 2\sqrt{2}$
- When $x=-1$ then $y^2=8$ so $y=\pm 2\sqrt{2}$

Solutions: $(1, 2\sqrt{2}), (1, -2\sqrt{2}), (-1, 2\sqrt{2}), (-1, -2\sqrt{2})$

Ex. 19.3 Solve (a) $xy=-4$ (hyperbola)

(b) $y=-x-2$ (line)

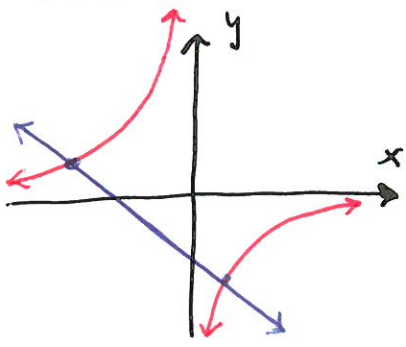
Solve (a) for y: $y=-\frac{4}{x}$

Sub into (b): $-4=-x^2-2x$ so $x^2+2x-4=0$

$$x=\frac{-2\pm\sqrt{5\cdot 4}}{2}=-1\pm\sqrt{5}$$

Sub x into (b):

$y=-1-\sqrt{5}$	$y=-1+\sqrt{5}$
$x=-1+\sqrt{5}$	$x=-1-\sqrt{5}$



Pre-Calc AB

Lesson 19

8/31/15

Factoring Exponentials

Ex. Simplify $\dfrac{x^{2a+3} - y^{2b}x^3}{x^{a+2} + y^b x^2}$

$$= \frac{x^3(x^{2a} - y^{2b})}{x^2(x^a + y^b)}$$

$$= \frac{x(x^a - y^b)(x^a + y^b)}{x^a + y^b}$$

$$= \boxed{x(x^a - y^b)}$$

Sum and Difference of Two Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Ex. 19.7 Factor: $8m^3y^6 + x^3$

$$= 2^3 m^3 (y^2)^3 + x^3$$

$$= (2my^2)^3 + x^3$$

$$= \boxed{(2my^2 + x)(4m^2y^4 - 2my^2x + x^2)}$$

Pre-Calc AB

Lesson 20

9/1/15

Agenda: 9/1/15

HW Leader:

Lesson 20

Two Special Triangles

Period 2: Richard K

Period 8: Lauren H.

★ Test 2 Tomorrow

- lessons 1-16
- problems

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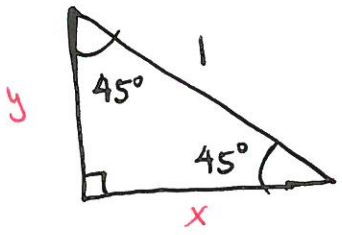
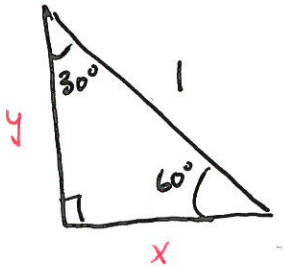
(1)



(2)

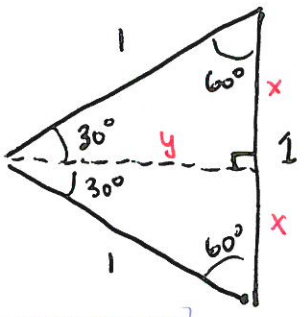


(1) $x = y$ So $1^2 = x^2 + x^2 = 2x^2$ thus $x = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$\sin(45°) = \frac{y}{1} = \frac{\sqrt{2}}{2}$

$\cos(45°) = \frac{x}{1} = \frac{\sqrt{2}}{2}$

$\tan(45°) = \frac{y}{x} = 1$



(2) So $x = \frac{1}{2}$ thus

$y^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$y = \frac{\sqrt{3}}{2}$

$\sin(30°) = x = \frac{1}{2}$

$\cos(30°) = y = \frac{\sqrt{3}}{2}$

$\tan(30°) = \frac{\sqrt{3}}{3}$

$\sin(60°) = y = \frac{\sqrt{3}}{2}$

$\cos(60°) = x = \frac{1}{2}$

$\tan(60°) = \sqrt{3}$