Caluclus AB NAME: _____

# MC Packet 7 - Particle Motion and Theorems

PERIOD: _____

**In-Class Together: Problems 1-6**

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① The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

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② Let $f$ be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. $f$ has at least 2 zeros.
II. The graph of $f$ has at least one horizontal tangent.
III. For some $c$, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

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③ If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number $c$ in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be $c$?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) $\pi$ (E) $\frac{3\pi}{2}$

④ A point moves in a straight line so that its distance at time $t$ from a fixed point of the line is $8t-3t^2$. What is the *total* distance covered by the point between $t=1$ and $t=2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

⑤ A point moves on the $x$-axis in such a way that its velocity at time $t$ $(t>0)$ is given by $v=\frac{\ln t}{t}$. At what value of $t$ does $v$ attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) $e$ (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for $v$.

⑥ The acceleration $\alpha$ of a body moving in a straight line is given in terms of time $t$ by $\alpha=8-6t$. If the velocity of the body is 25 at $t=1$ and if $s(t)$ is the distance of the body from the origin at time $t$, what is $s(4)-s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

(7) Let $f$ be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of $x$ between $a$ and $b$?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

(8) A particle moves along the $x$-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of $t$ is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

(9) What is the average value of $y$ for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) $-6$ (B) $-2$ (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

(10) The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$

(B) $(1,1)$

(C) $\left(2,\sqrt{2}\right)$

(D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$

(E) None of the above

(11) The position of a particle moving along a straight line at any time $t$ is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

(12) A particle moves along the $x$-axis so that its position at time $t$ is given by $x(t) = t^2 - 6t + 5$. For what value of $t$ is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(13) The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |

(14) The function $f$ is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

(15) Let $f$ be the function given by $f(x)=x^3-3x^2$. What are all values of $c$ that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

(16) The velocity of a particle moving on a line at time $t$ is $v=3t^{\frac{1}{2}}+5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t=0$ to $t=4$?

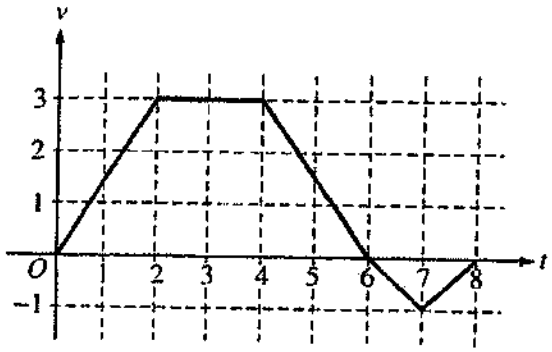
(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

(17) A particle moves in a straight line with velocity $v(t)=t^2$. How far does the particle move between times $t=1$ and $t=2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

(18) If the position of a particle on the $x$-axis at time $t$ is $-5t^2$, then the average velocity of the particle for $0\le t\le 3$ is

(A) $-45$ (B) $-30$ (C) $-15$ (D) $-10$ (E) $-5$




(19) A bug begins to crawl up a vertical wire at time $t = 0$. The velocity $v$ of the bug at time $t$, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of $t$ does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

(20) A particle moves along the $x$-axis so that at any time $t$ its position is given by $x(t) = te^{-2t}$. For what values of $t$ is the particle at rest?

(A) No values (B) 0 only (C) $\frac{1}{2}$ only (D) 1 only (E) 0 and $\frac{1}{2}$

(21) At $t = 0$ a particle starts at rest and moves along a line in such a way that at time $t$ its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

(22) The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

(23) The average value of $f(x) = x^2\sqrt{x^3 + 1}$ on the closed interval $[0, 2]$ is

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26