Caluclus AB NAME: _____

MC Packet 6 - Riemann Sums, Areas, and Volumes PERIOD: _____

In-Class Together: Problems 1-5

x	2	5	7	8
$f(x)$	10	30	40	20

(1) The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

(2) The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

(3) The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

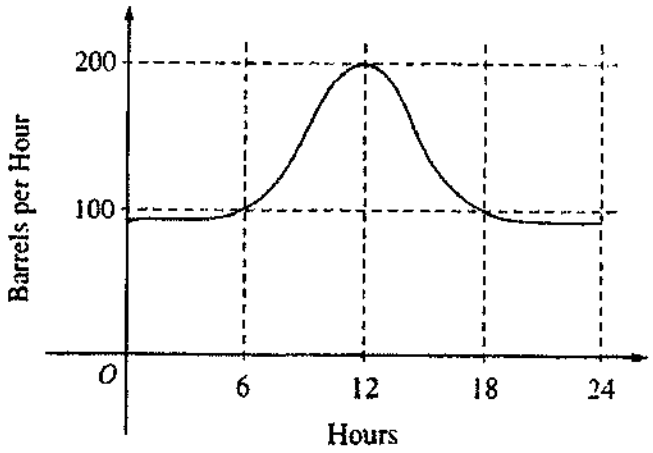
(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

(4) If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π

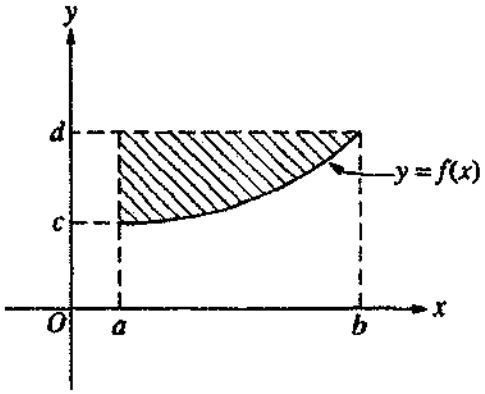
(5) The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1 - e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1 - e^{-3}$



(6) The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800



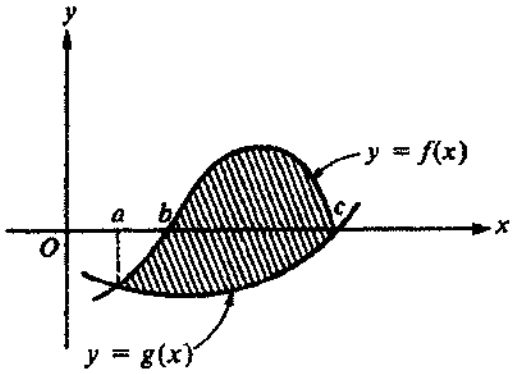
(7) Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$ (B) $\int_a^b (d - f(x))dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

(8) The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π



(9) The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c \left(|f(x)|-|g(x)|\right)dx$

(B) $\int_b^c f(x)\,dx-\int_a^c g(x)\,dx$

(C) $\int_a^c \left(g(x)-f(x)\right)dx$

(D) $\int_a^c \left(f(x)-g(x)\right)dx$

(E) $\int_a^b \left(g(x)-f(x)\right)dx+\int_b^c \left(f(x)-g(x)\right)dx$

(10) The area of the region enclosed by the graph of $y=x^2+1$ and the line $y=5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

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(11) What is the area of the region completely bounded by the curve $y=-x^2+x+6$ and the line $y=4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

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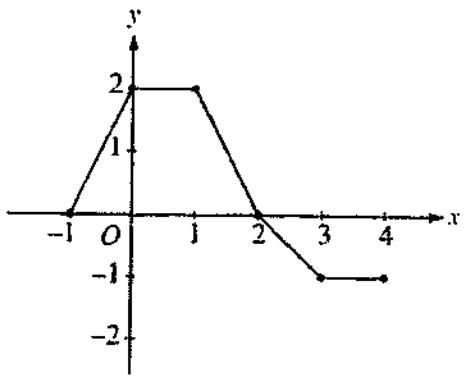
(12) The region in the first quadrant bounded by the graph of $y=\sec x$, $x=\frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi-1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$



(13) The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

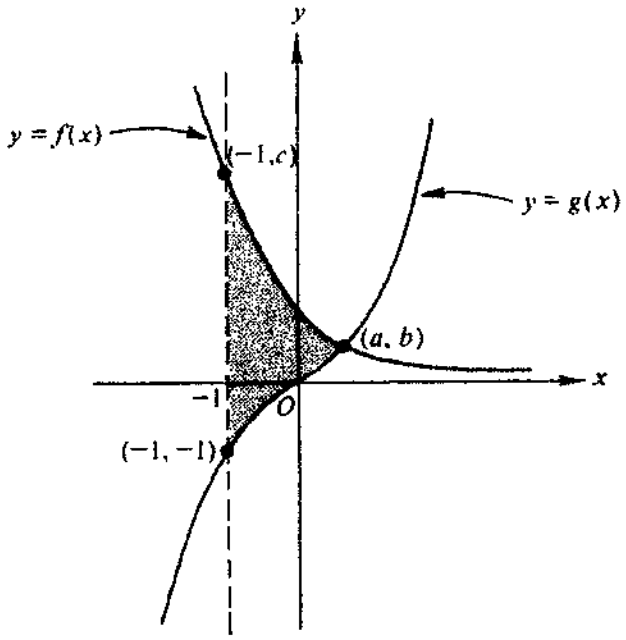
(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

(14) The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$

(15) The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\,\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$



(16) The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a, b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

(A) $\int_0^a (f(x) - g(x))dx + \int_{-1}^0 (f(x) + g(x))dx$

(B) $\int_{-1}^b g(x)dx + \int_b^c f(x)dx$

(C) $\int_{-1}^c (f(x) - g(x))dx$

(D) $\int_{-1}^a (f(x) - g(x))dx$

(E) $\int_{-1}^a (|f(x)| - |g(x)|)dx$

(17) The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

(18) A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

(19) The area of the region enclosed by the curve $y=\frac{1}{x-1}$, the x-axis, and the lines $x=3$ and $x=4$ is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

(20) The area of the region bounded by the lines $x=0$, $x=2$, and $y=0$ and the curve $y=e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

(21) The region enclosed by the graph of $y=x^2$, the line $x=2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$