

## In-Class Together: Problems 1-6

① What are all values of  $x$  for which the function  $f$  defined by  $f(x) = (x^2 - 3)e^{-x}$  is increasing?

- (A) There are no such values of  $x$ .
  - (B)  $x < -1$  and  $x > 3$
  - (C)  $-3 < x < 1$
  - (D)  $-1 < x < 3$
  - (E) All values of  $x$
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② At  $x = 0$ , which of the following is true of the function  $f$  defined by  $f(x) = x^2 - e^{-2x}$ ?

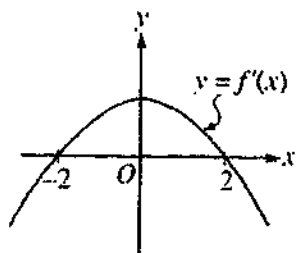
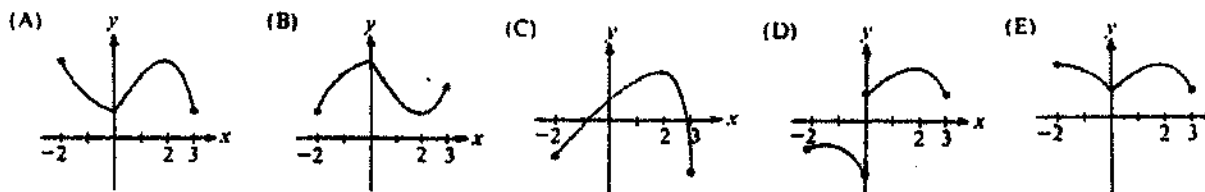
- (A)  $f$  is increasing.
  - (B)  $f$  is decreasing.
  - (C)  $f$  is discontinuous.
  - (D)  $f$  has a relative minimum.
  - (E)  $f$  has a relative maximum.
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③ For what value of  $k$  will  $x + \frac{k}{x}$  have a relative maximum at  $x = -2$ ?

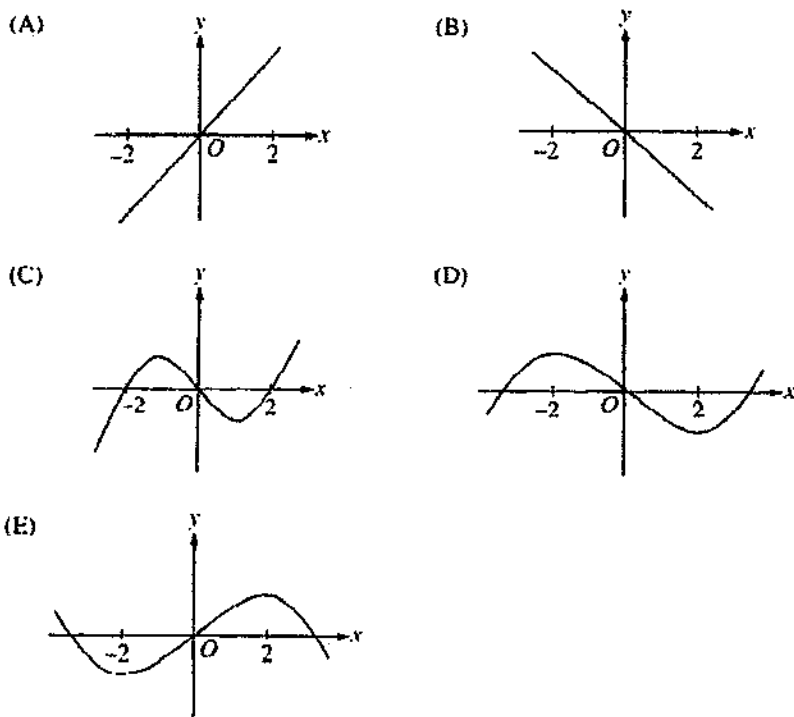
- (A)  $-4$
  - (B)  $-2$
  - (C)  $2$
  - (D)  $4$
  - (E) None of these
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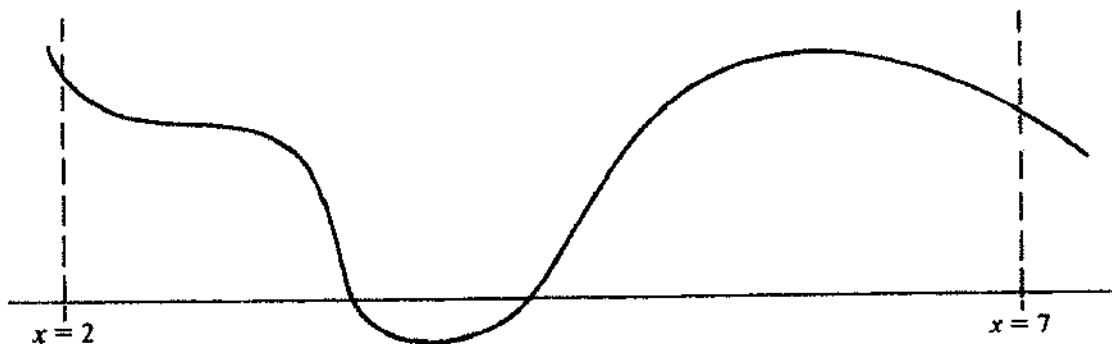
- ④ If  $f''(x) = x(x+1)(x-2)^2$ , then the graph of  $f$  has inflection points when  $x =$
- (A)  $-1$  only (B)  $2$  only (C)  $-1$  and  $0$  only (D)  $-1$  and  $2$  only (E)  $-1, 0,$  and  $2$  only

- ⑤ Let  $f$  be a function that is continuous on the closed interval  $[-2, 3]$  such that  $f'(0)$  does not exist,  $f'(2) = 0$ , and  $f''(x) < 0$  for all  $x$  except  $x = 0$ . Which of the following could be the graph of  $f$ ?



- ⑥ The graph of the derivative of  $f$  is shown in the figure above. Which of the following could be the graph of  $f$ ?





7 The graph of  $y = f(x)$  on the closed interval  $[2, 7]$  is shown above. How many points of inflection does this graph have on this interval?

- (A) One      (B) Two      (C) Three      (D) Four      (E) Five

8 The graph of  $y = 3x^4 - 16x^3 + 24x^2 + 48$  is concave down for

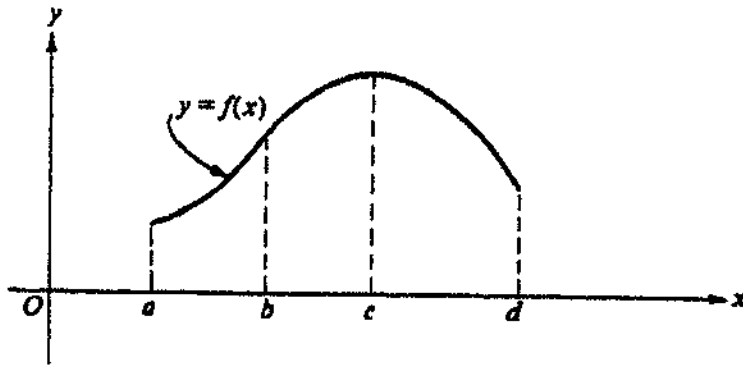
- (A)  $x < 0$   
 (B)  $x > 0$   
 (C)  $x < -2$  or  $x > -\frac{2}{3}$   
 (D)  $x < \frac{2}{3}$  or  $x > 2$   
 (E)  $\frac{2}{3} < x < 2$

9 If  $y = 2x - 8$ , what is the minimum value of the product  $xy$ ?

- (A) -16      (B) -8      (C) -4      (D) 0      (E) 2

10 The graph of  $y = 5x^4 - x^5$  has a point of inflection at

- (A) (0,0) only      (B) (3,162) only      (C) (4,256) only  
 (D) (0,0) and (3,162)      (E) (0,0) and (4,256)



- (11) The graph of  $y = f(x)$  is shown in the figure above. On which of the following intervals are  $\frac{dy}{dx} > 0$  and  $\frac{d^2y}{dx^2} < 0$ ?

- I.  $a < x < b$
- II.  $b < x < c$
- III.  $c < x < d$

- (A) I only      (B) II only      (C) III only      (D) I and II      (E) II and III

- (12) The function  $f$  is given by  $f(x) = x^4 + x^2 - 2$ . On which of the following intervals is  $f$  increasing?

- (A)  $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
- (B)  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
- (C)  $(0, \infty)$
- (D)  $(-\infty, 0)$
- (E)  $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

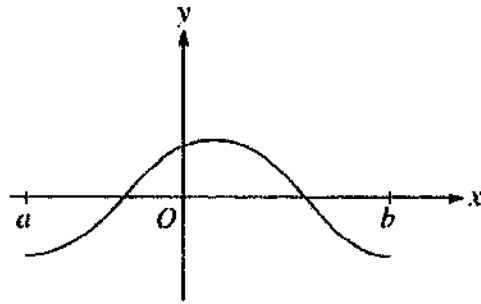
- (13) The derivative of  $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$  attains its maximum value at  $x =$

- (A)  $-1$       (B)  $0$       (C)  $1$       (D)  $\frac{4}{3}$       (E)  $\frac{5}{3}$

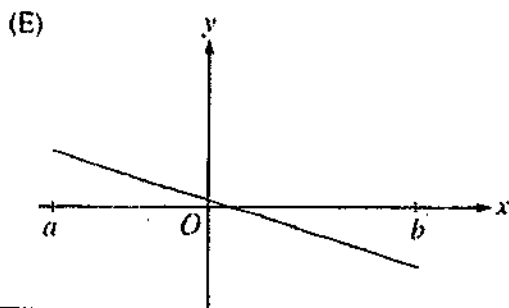
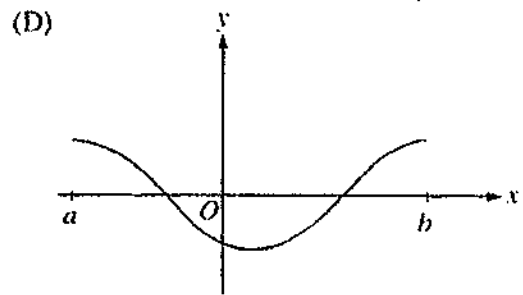
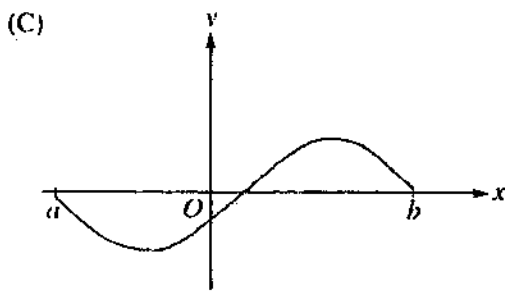
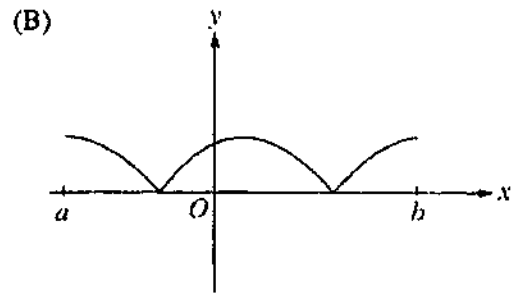
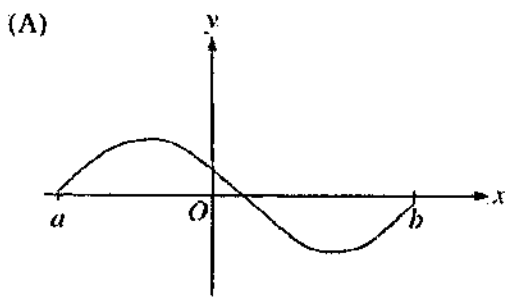
- (14) At what values of  $x$  does  $f(x) = 3x^5 - 5x^3 + 15$  have a relative maximum?

- (A)  $-1$  only      (B)  $0$  only      (C)  $1$  only      (D)  $-1$  and  $1$  only      (E)  $-1, 0$  and  $1$

Homework: Problems 15-28



(15) The graph of  $f$  is shown in the figure above. Which of the following could be the graph of the derivative of  $f$ ?



(16) Let  $f$  be a function defined for all real numbers  $x$ . If  $f'(x) = \frac{4-x^2}{x-2}$ , then  $f$  is decreasing on the interval

- (A)  $(-\infty, 2)$       (B)  $(-\infty, \infty)$       (C)  $(-2, 4)$       (D)  $(-2, \infty)$       (E)  $(2, \infty)$

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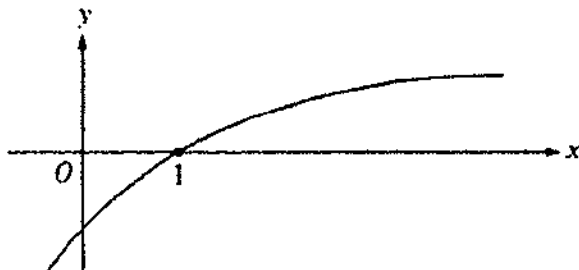
At what value of  $x$  does the graph of  $y = \frac{1}{x^2} - \frac{1}{x^3}$  have a point of inflection?

- (A) 0      (B) 1      (C) 2      (D) 3      (E) At no value of  $x$

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The absolute maximum value of  $f(x) = x^3 - 3x^2 + 12$  on the closed interval  $[-2, 4]$  occurs at  $x =$

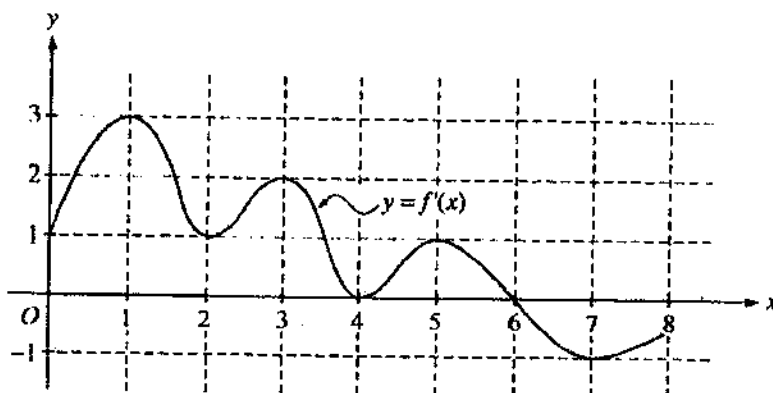
- (A) 4      (B) 2      (C) 1      (D) 0      (E) -2



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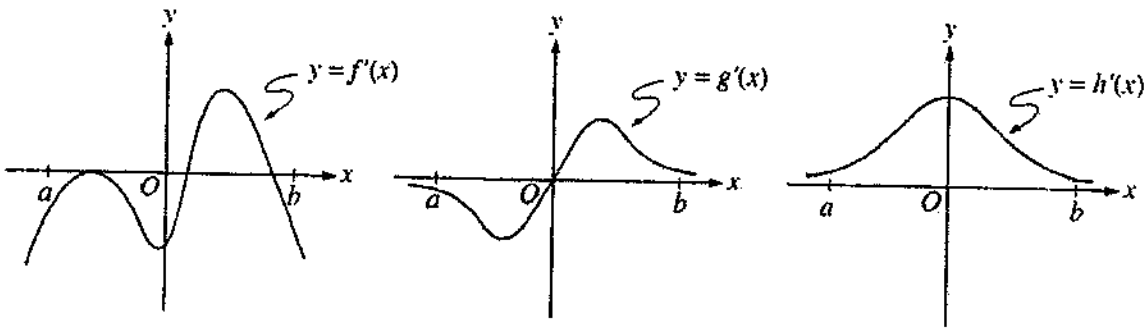
The graph of a twice-differentiable function  $f$  is shown in the figure above. Which of the following is true?

- (A)  $f(1) < f'(1) < f''(1)$   
 (B)  $f(1) < f''(1) < f'(1)$   
 (C)  $f'(1) < f(1) < f''(1)$   
 (D)  $f''(1) < f(1) < f'(1)$   
 (E)  $f''(1) < f'(1) < f(1)$



20 The function  $f$  is defined on the closed interval  $[0, 8]$ . The graph of its derivative  $f'$  is shown above. How many points of inflection does the graph of  $f$  have?

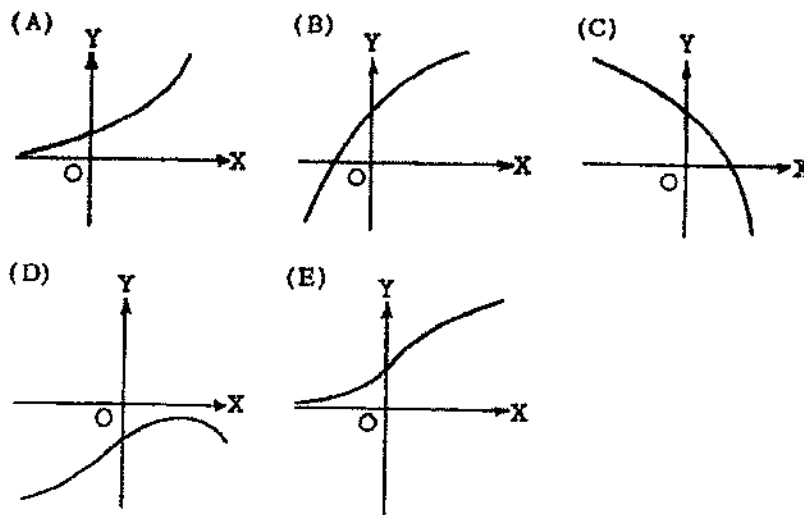
- (A) Two  
 (B) Three  
 (C) Four  
 (D) Five  
 (E) Six



21 The graphs of the derivatives of the functions  $f$ ,  $g$ , and  $h$  are shown above. Which of the functions  $f$ ,  $g$ , or  $h$  have a relative maximum on the open interval  $a < x < b$ ?

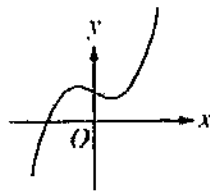
- (A)  $f$  only
- (B)  $g$  only
- (C)  $h$  only
- (D)  $f$  and  $g$  only
- (E)  $f$ ,  $g$ , and  $h$

22 If  $y$  is a function of  $x$  such that  $y' > 0$  for all  $x$  and  $y'' < 0$  for all  $x$ , which of the following could be part of the graph of  $y = f(x)$ ?

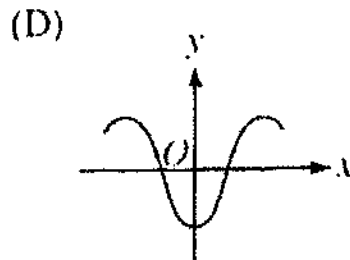
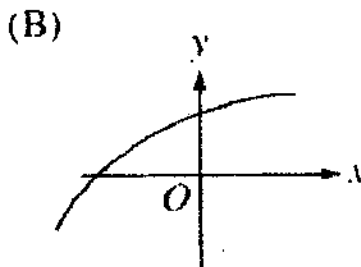
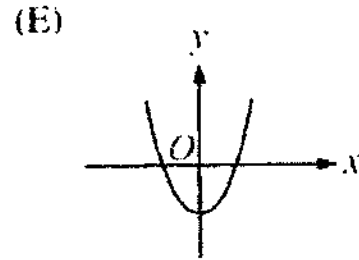
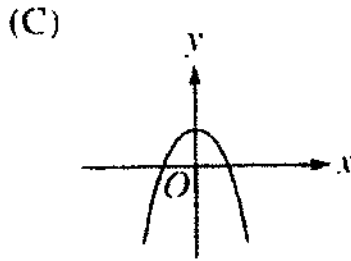
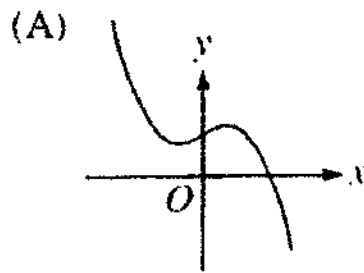


23 If  $f(x) = \frac{\ln x}{x}$ , for all  $x > 0$ , which of the following is true?

- (A)  $f$  is increasing for all  $x$  greater than 0.
- (B)  $f$  is increasing for all  $x$  greater than 1.
- (C)  $f$  is decreasing for all  $x$  between 0 and 1.
- (D)  $f$  is decreasing for all  $x$  between 1 and  $e$ .
- (E)  $f$  is decreasing for all  $x$  greater than  $e$ .



24 The graph of  $y = h(x)$  is shown above. Which of the following could be the graph of  $y = h'(x)$ ?



25 If  $g$  is a differentiable function such that  $g(x) < 0$  for all real numbers  $x$  and if  $f'(x) = (x^2 - 4)g(x)$ , which of the following is true?

- (A)  $f$  has a relative maximum at  $x = -2$  and a relative minimum at  $x = 2$ .  
 (B)  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .  
 (C)  $f$  has relative minima at  $x = -2$  and at  $x = 2$ .  
 (D)  $f$  has relative maxima at  $x = -2$  and at  $x = 2$ .  
 (E) It cannot be determined if  $f$  has any relative extrema.

26 How many critical points does the function  $f(x) = (x+2)^5(x-3)^4$  have?

- (A) One      (B) Two      (C) Three      (D) Five      (E) Nine

27 The graph of  $y = \frac{-5}{x-2}$  is concave downward for all values of  $x$  such that

- (A)  $x < 0$       (B)  $x < 2$       (C)  $x < 5$       (D)  $x > 0$       (E)  $x > 2$

28 What is the minimum value of  $f(x) = x \ln x$ ?

- (A)  $-e$       (B)  $-1$       (C)  $-\frac{1}{e}$       (D)  $0$       (E)  $f(x)$  has no minimum value.